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Intertemporal risk parity: a constant volatility framework for factor investing

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Intertemporal risk parity is a strategy that rebalances risky assets and cash in order to target a constant level of ex ante risk over time. When applied to equities and compared with a buy-and-hold portfolio it is known to improve the Sharpe ratio and reduce drawdowns. We apply intertemporal risk parity strategies to factor investing, namely value and momentum investing in equities, government bonds and foreign exchange. Value and momentum factors generate a premium which is traditionally captured by dollar-neutral long–short portfolios rebalanced every month to take into account changes in stock, bond or foreign exchange factor exposures and keep leverage constant. An intertemporal risk parity strategy rebalances the portfolio to the level of leverage required to target a constant ex ante risk over time. Value and momentum risk-adjusted premiums increase, sometimes significantly, when an intertemporal risk parity strategy is applied. Volatility clustering and fat tails are behind this improvement of risk-adjusted premiums. Drawdowns are, however, not smoothed when applying the strategy to factor investing. The benefits of the intertemporal risk parity strategy are more important for factor premiums with strong negative relationship between volatility and returns, strong volatility clustering and fat tails.

1 INTRODUCTION

The literature on factor investing is extensive and dates back many years. Basu (1977) is often given the credit for uncovering the value premium, even if the idea of value investing is older and has been known about at least since the work of Graham (1949). Value stocks are said to be trading at prices too low for their level of earnings, book value or cashflow and have historically delivered returns above what would be expected from their level of risk. Banz (1981) discovered the size premium arising from investing in small-capitalization stocks. Small-capitalization stocks have also been paying higher returns than expected from their level of risk over time. Jagadeesh and Titman (1993) uncovered the momentum premium arising from investing in stocks with the fastest past trends over horizons of about twelve months. On average, these stocks have also been paying returns above what would be expected considering their level of risk. Low-risk premium arising from investing in stocks with the lowest volatility or lowest beta was uncovered by Haugen and Heins (1972). Low-volatility stocks have been paying much higher returns than expected considering their level of risk and even manage to outperform the riskier stocks in a number of markets (Haugen and Baker 2012).

Factor investing began attracting attention with the proposal of Fama and French (1992, 1993) of a three-factor model for assessing the alpha in portfolios by estimating not only the exposure to the market portfolio, but also the exposure to value and size premiums. Charhart (1997) extended the Fama–French model to include the momentum premium and Scherer (2011) proposed the addition of low-risk factors to measure exposure to the low-risk premium.

Bender *et al* (2014) showed recently that the performance of traditional portfolios could be split into beta, alpha and factor-premium contributions. They explain that factor-premium contributions are the portion of alpha that can be captured via systematic investment strategies. Indeed, the key reason to separate alpha-generated from factor exposures from other alpha sources is the fact that factor premiums can be accessed through relatively simple systematic strategies.

Recent research also demonstrates that the increasingly popular smart-beta equity indexes earn their excess returns from exposures to factors. Scherer (2011) showed that minimum variance is essentially exposed to the low-risk premium. De Carvalho *et al* (2012) extended his work to show that maximum diversification as introduced by Choueifaty and Coignard (2008) is also strongly exposed to the low-risk premium and that risk-parity strategies, as first introduced by Dalio (2005), generate strong exposures to the size, low-risk and value premiums when applied to stock universes. Finally, Chow *et al* (2011) showed that fundamental indexing, another type of smart-beta equity indexation, is essentially exposed to value premium.

Asness *et al* (2013) showed that momentum and value premiums are not exclusive of the equity markets but can also be found in other asset classes such as government bonds, foreign exchange or commodities. They also demonstrated that combinations of factor premiums result in better performances than exposures to a single factor premium. This adds to the evidence previously found by Qian *et al* (2009) that value premium can be captured not only in equities but also in government bonds and foreign exchange.

Capturing factor premiums requires an active strategy that rebalances the portfolio to adjust for factor exposures. Stock exposures to factors change over time and the active strategy must trade accordingly. The most common strategy is to create a long–short portfolio which invests in the stocks with the strongest exposure to the factor and sells short stocks with the weakest exposures. Stocks are equally weighted (and sometimes market-cap weighted) in both the long and short portfolios and the long–short portfolios are periodically rebalanced to a given predefined level of leverage relative to assets under management which is kept constant over time. This factor strategy can also be applied to other asset classes.

As Asness *et al* (2013) pointed out, there is not one unique active strategy but many different systematic strategies which can be used to capture factor premiums. Neither the volatility of stocks, nor correlations between pairs of stocks, is constant over time. Thus, the constant leveraging of a long–short portfolio is sure to have variable risk over time. Rather than rebalancing to a constant level of leverage, Asness *et al* (2013) propose instead that the long–short portfolios should be rebalanced toward the leverage which is required to target a given predefined level of *ex ante* risk. They claim that the strategy which rebalances toward constant *ex ante* risk delivers better risk-adjusted returns than the strategy which rebalances toward constant leverage, at least before transaction costs and market impact are considered. However, they fall short of explaining why risk-adjusted returns improve when a constant level of risk is targeted. Barroso and Santa-Clara (2013) also suggest that momentum premium in equity markets is better captured if the portfolio is rebalanced toward a constant risk target. They relate the improvement in momentum risk-adjusted returns to the predictability of the momentum risk and also highlight that the management of risk seems to reduce exposure to fat tails.

Perchet *et al* (2014) recently revealed that the strategy which rebalances the weight of an asset class toward a constant level of *ex ante* risk, an intertemporal risk-parity strategy, generates superior risk-adjusted returns when compared to a buy-and-hold portfolio, at least before transaction costs and market impact are considered. They show that the improvement in risk-adjusted returns is larger for risky asset classes such as emerging-market equities or high yield. Developed equities and commodities also have higher risk-adjusted returns when managed at a constant risk, however, for investment-grade corporate bonds and in particular for government bonds, the improvement in risk-adjusted returns is negligible. Perchet *et al* (2014) provide a number of explanations for the improvement in risk-adjusted returns in constant risk strategies. First they highlight that in a world where returns follow normal distributions, the constant risk strategy would be equivalent to the buy-and-hold strategy since the volatility of asset returns would be constant over time. The added value of the constant risk strategy comes from the fact that although volatility is not constant (and not even observable) in the real world, volatility of financial assets tends to form clusters, a property which tends to be stronger in risky assets. Clustering of volatility makes volatility predictable to some extent: when volatility is low it is more likely to remain low and when is high it tends to remain high. Constant volatility strategies thus reduce the exposure to financial assets in periods of high volatility and increase the exposure in periods of high volatility in such a way that *ex post* volatility is successfully kept at reasonably constant levels as targeted. If average returns were constant, that would mean increasing the exposure to the financial asset in periods of higher risk-adjusted returns and reducing it in periods of lower risk-adjusted returns. But as shown by Perchet *et al* (2014), for asset classes such as equities and high yield there is even a negative correlation between returns and volatility which strengthens the effect. Fat tails, which, as revealed by Cont (2000), are present in the distribution of returns of most asset classes, tend to be stronger in periods of higher volatility. Fat tails are then smoothed out by the reduction of the exposure to the financial assets in periods of higher volatility with a positive impact on risk measures such as the value at risk (VaR), conditional value at risk (CVaR) or the size of drawdowns.

In this paper, we extend the approach of Perchet *et al* (2014) to factor investing. We focus on value and momentum factors in equity markets, government bond markets and foreign exchange markets. We give evidence of volatility regimes with different average returns in the time series of the returns of the long–short portfolios behind the systematic strategies constructed to capture factor premiums. A negative relationship between volatility and returns is a particularly important feature in the distribution of returns of strategies designed to capture some factor premiums, eg, equity momentum, although this negative correlation is less strong for value-capture strategies in equity and government markets. We also find that volatility clustering is strong in the factor returns. Volatility clustering has an important impact in explaining the superior information ratio of strategies designed to capture factor premiums which are managed by targeting a constant level of *ex ante* risk over those targeting constant leverage. In all, for each factor premium considered we estimate the contribution of a number of possible effects to the improved risk-adjusted return of constant risk strategies designed to capture those factor premiums.

First, we present the methodology used to build strategies to capture momentum and value premiums in equity, government bond and foreign-exchange markets. Next, we start with an analysis of volatility regimes in the time series of factor returns and the relationship between average returns and volatility in each regime. We then discuss the evidence and importance of volatility clustering and analyze the relationship between the strategy returns and volatility. We apply intertemporal risk-parity strategies and investigate the relationship between the volatility properties and the improvement of the information ratio observed for those factor strategies. At the end, we discuss the impact of the choice of the target risk of the intertemporal risk parity strategy.

2 DATA

In this section we provide details on the data and investment universe used in the strategies designed to capture factor premiums in the three assets classes considered. Unless otherwise stated, all data has been obtained from Bloomberg.

2.1 Government bonds

We used short rates, five-year government bond yields and ten-year government bond yields for ten countries: Australia, Canada, Germany, Japan, Denmark, Norway, Sweden, Switzerland, the UK and the US. The data used always covered a minimum of seven countries. Data for all countries was only available from April 1990. We built the time series of daily returns for currency-hedged total-return indexes, ie, coupons reinvested with a constant maturity of ten years. The five-year yields were used to include the roll-down effect.

2.2 Foreign exchange

We use the exchange spot rates against the US dollar and interbank rates for ten countries: Australia, Canada, Germany or the eurozone after 1999, Japan, New Zealand, Norway, Sweden, Switzerland, the UK and the US. The data covered a minimum of seven exchange rates. Data for exchange rates was available from November 1995.

2.3 Equities

We used the daily data from Kenneth French's website,[1] all in US dollars and with dividends reinvested. We focused on two strategies:

(i) the high-minus-low (HML), which captures value premium, and

(ii) the momentum (Mom), which captures momentum premium.

Data was available from January 1980. The universe includes all NYSE, AMEX and NASDAQ firms.

[1] URL: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

3 METHODOLOGY

In this section we discuss the methodology used in the construction of the systematic strategies designed to capture factor premiums. Different approaches for capturing value or momentum premiums can be found in the academic literature. Our approach was to adopt the most common value and momentum factor portfolios and compare the performance of strategies which invest in such portfolios while targeting constant leverage against those targeting constant *ex ante* risk.

3.1 Government bonds

Jegadeesh (1990) showed that, in the short term, mean reversion bottom-up strategies are profitable, ie, buying the recent losers and selling the recent winners. For that reason, momentum premium is usually captured using the returns over the past eleven months calculated one month before forming the portfolio. Since there is no evidence that short-term mean reversion is a profitable strategy in government bond markets, we used the cumulative returns of the total-return index for each country from the past twelve months (see Jegadeesh and Titman 1993; Asness 1994; Fama and French 1996; Grinblatt and Moskowitz 2004). Thus, contrary to Asness *et al* (2013) and what is common for equities, we did not skip the most recent month when building the momentum-factor portfolios for government bonds.

For value premium we used the slope of the yield curve between the ten-year bond yields and cash rates to build the portfolio. Countries are ranked according to this yield curve slope and the portfolio invests in the countries with the steepest slopes and sells short the countries with the flattest or even inverted slopes. This approach was proposed by Qian *et al* (2009).

In both cases, the portfolio is rebalanced every month. Daily returns of the strategy were calculated taking into account portfolio drift.

3.2 Foreign exchange

Since there is no evidence of short-term mean reversion in the foreign-exchange markets, we used a strategy to capture momentum premium equivalent to that of government bonds. We calculated the twelve-month cumulative returns of forward contracts, eg, a dollar-neutral portfolio that borrows in US dollars and buys foreign currency to be invested at foreign cash rates. For the US dollar, the performance of this portfolio is obviously zero at all times.

In order to capture the value premium, we used the strategy proposed by Qian *et al* (2009), which is commonly known as the carry strategy. The portfolio opens long positions in forward contracts of currencies of countries with the highest cash rates and opens short positions in forward contracts of currencies with the lowest cash rates. We did not follow Asness *et al* (2013) who proposed an alternative approach to capture value premium based on the last five-year performance of the different exchange rates.

3.3 Equities

We used the return time series provided by Kenneth French for the value (HML) and momentum (Mom) portfolios available from his website.[2] The strategy designed to capture momentum premium is based on returns over the last ten months estimated two months prior to portfolio formation. In this way, the short-term mean reversion will not have an impact on the momentum strategy. The selection of stocks is based on sub-universes split according to market capitalization of stocks in order to avoid creating a size bias in the strategy. The portfolios are long stocks with the strongest momentum and sell short stocks with the poorest momentum. Only large and small-capitalization stocks are considered for the strategy. Mid-cap stocks are always excluded.

For value premium, stocks are also split according to size and then selected according to the book-to-market inside these sub-buckets. The factor is based only on small and large-cap stocks, ie, mid-cap stocks are not used in the construction of the portfolios.

3.4 Portfolio construction

The value and momentum-factor strategies for government bonds and for foreign exchange have the same notional at risk in both the long and short legs of the portfolio and the long–short portfolios were built using a methodology similar to that proposed by Asness *et al* (2009). For government bonds, the weight w_{it}^{factor} of each country i in the long–short portfolios at time t is

$$W_{it}^{\text{factor}} = C_t \left[\text{rank}(\text{factor}_{it}) - \sum_i \text{rank}(\text{factor}_{it})/N \right], \tag{3.1}$$

where the sums run for all countries $i = 1$ to N (in this paper $N = 10$ for both government bonds and for foreign exchange). For value, factor_{it} is the slope of yield curve of the country i at time t and for momentum the twelve-month returns to a forward contract against the US dollar. The function rank returns the ranking of factor values in the investment universe. The weights are thus proportional to the cross-sectional ranking of the factor centered about the cross-sectional average ranking. The long–short portfolio above sums to zero at the time of formation. This methodology to build portfolios takes into account the strength of asset factor exposures when building the long–short portfolio.

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[2] URL: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

Countries with a steeper slope or stronger momentum will be assigned a larger weight in the long portfolio, countries with the most inverted slope (or least steep slope) or poorest momentum will be assigned a larger weight in the short portfolio. We have chosen to scale the long–short portfolio to US\$1 long notional exposure and to US\$1 short notional exposure. The return of the proposed strategy in a given period is estimated from

$$r_t = \sum_{i=1}^{N} W_{it}^{\text{factor}} r_{it}. \tag{3.2}$$

As mentioned, the portfolios are rebalanced every month. The distribution of returns of these factor strategies does not follow a normal distribution and does not appear to have constant volatility. We can thus apply an intertemporal risk-parity strategy by adjusting C_t in (3.1) over time in order to rebalance the portfolio to a predefined constant level of *ex ante* volatility. We will examine the returns of these factor strategies in both cases: keeping C_t constant and adjusting C_t to target constant *ex ante* volatility.

4 VOLATILITY PROPERTIES AND INTERTEMPORAL RISK PARITY STRATEGIES

We now focus on the different properties of the volatility of returns generated by these factor strategies.

4.1 Evidence of volatility regimes

We start with an analysis of the time series of daily factor returns assuming C_t is constant and equal to 1. We fit a two-state hidden Markov model (HMM) to the factor returns using maximum likelihood estimation (see Hamilton (1994) for more details). This allows us to check whether the factor returns can be classified into two regimes, one regime with higher volatility and one with lower volatility, and estimate the average return and volatility of the underlying Gaussian distribution of each of those regimes. One output of the HMM is a probability of transition matrix. This matrix gives the probability that given today's regime the return tomorrow will stay in the same regime and the probability that given today's regime the return tomorrow will change regime. The results from the application of the HMM are shown in Table 1 on page 10. All factors have two separate regimes. For all the factors considered, there is a higher volatility regime with a lower average return and a lower volatility regime with a higher average return.

In addition, the regimes are significant because the probability of staying in the current regime is high and the differences in the volatility of each regime are significant. In other words, factor returns can be thought as being in one of two hidden regimes with significantly different volatility and when a factor return is identified with one regime then the factor return on the next day tends to be identified with the same regime.

Factor returns tend to be drawn more often from the lower volatility hidden regime than from the higher volatility regime. Foreign exchange momentum has the smallest number of factor returns identified with the higher volatility regime, 19.5% of daily returns, while government bonds momentum has the largest, 37.2% of daily returns.

The average returns of the lower volatility regime are higher than the average returns of the higher volatility regimes. With the exception of equity value, the difference in risk-adjusted returns between the two regimes is significant with the average returns of the higher volatility regime either negative or negligible and the average returns of the lower volatility regimes positive and typically high for the level of volatility.

The large differences between the average volatility and return in the two regimes of a given factor and the large probability of staying in the current regime are supportive of the application of intertemporal risk strategies, ie, adjusting C_t to target constant *ex ante* volatility as a means for improving risk-adjusted returns. The smaller differences between the volatility of regimes in government bond factors and the small difference in information ratio of regimes in equity value anticipates weaker improvement in the information ratio of a strategy targeting a constant volatility over a strategy using constant leverage for these.

4.2 Volatility clustering: α and fat tails

We shall first consider volatility clustering and fat tails. Generalized autoregressive conditional heteroscedasticity (GARCH) models with noise drawn from a Student t distribution capture both these properties. The formulation of this particular GARCH model was proposed by Bollerslev (1986):

$$\left.\begin{aligned} r_t &= \mu + \sigma z_t \quad \text{with } z_t \sim t(0, 1, \nu), \\ \sigma_t^2 &= \omega + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2. \end{aligned}\right\} \tag{4.1}$$

The return r_t at time t is a function of the volatility σ_t and the μ. The noise z_t is drawn from a Student t distribution with ν degrees of freedom. The larger the β relative to α, the more stable the volatility is over time, and the larger the α, the more the volatility clusters. $\alpha + \beta < 1$ must hold for stationarity. ω represents a long-term constant volatility term. The smaller the parameter ν, the larger the probability of extreme events, which are more frequent in higher-volatility regimes.

In Table 2 on page 11 we show the estimated parameters of a GARCH model with Student t noise as in (4.1) using maximum likelihood approaches from the historical returns of constant leverage factor strategies in the period indicated. We find that $\alpha + \beta$ is always below but close to 1, which indicates that the volatility is essentially

TABLE 1 Estimated parameters of a two regime hidden Markov model applied to the time series of daily factor returns.

(a) Regime 1

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
Annualized excess return over cash (%)	18.0	2.8	10.6	12.9	4.0	4.9
Annualized volatility (%)	7.1	5.6	6.2	5.9	3.8	4.4
Information ratio	2.55	0.50	1.70	2.20	1.05	1.11
Percentage of observations identified with regime	74.8	75.0	80.5	78.7	62.8	76.9
Probability that return at $t+1$ is in this regime (%)	99.0	99.4	97.9	98.1	98.0	97.1

(b) Regime 2

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
Annualized excess return over cash (%)	−16.1	6.5	−24.6	−21.0	−5.2	0.5
Annualized volatility (%)	24.9	16.5	15.5	14.6	8.1	10.0
Information ratio	−0.65	0.40	−1.59	−1.44	−0.65	0.05
Percentage of observations identified with regime	25.2	25.0	19.5	21.3	37.2	23.1
Probability that return at $t+1$ is in this regime (%)	97.2	98.1	92.2	93.7	96.5	92.2

"Mom" is momentum. The parameters were obtained from maximum likelihood estimation using data from January 1, 1990 to December 31, 2013.

explained by past volatility and new events. Momentum and value premiums in equities have a large α, indicating strong clustering of volatility. We also find a significant α in foreign-exchange value and momentum. The α for value and momentum premium in government bonds is much smaller. The Student t ν tends to be quite large for all factor strategies except for value in equities. This indicates strong fat tails for all factors and less strong for value equities. Factor investing seems to be exposed to extreme events. As we shall see, application of an intertemporal risk-parity strategy will smooth those events.

TABLE 2 Estimated parameters of a GARCH model with Student t noise for the different factor strategies at a constant leverage.

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
ω	2.3E-07	2.6E-07	5.8E-07	6.2E-07	1.1E-07	9.6E-08
α (%)	9.2	9.5	7.6	7.8	5.1	5.5
(t-stat of α)	11.8	10.0	0.1	10.7	9.3	10.2
β	90.5 (%)	89.7	90.6	90.2	94.1	94.0
(t-stat of β)	110.1	84.8	0.7	91.9	140.9	150.0
$\alpha + \beta$	99.7 (%)	99.3	98.3	98.0	99.2	99.5
Student t ν	9.6	11.2	5.9	6.1	6.0	6.7
(t-stat of Student t)	9.8	7.6	0.5	16.9	16.6	15.9

"Mom" is momentum. The estimation of parameters uses data from January 1, 1990 to December 31, 2013.

4.3 Gain and loss asymmetry: the leverage effect and negative skew

We now look at two other volatility properties which are often highlighted in literature for asset-class returns. The first is negative skew, ie, the fact that the prices of financial assets usually increase slowly but tend to fall suddenly with extreme negative return. The second is what is known as the leverage effect, ie, the fact that negative returns in asset prices tend to induce higher volatility, suggesting that returns may show a negative correlation with volatility. In other words, periods of low volatility tend to be associated with positive performance and periods of high volatility tend to be associated with low or negative returns.

Glosten *et al* (1993) proposed a modified GARCH model, known as the Glosten–Jagannathan–Runkle GARCH (GJR-GARCH) model, which introduces the relationship between return and volatility:

$$\left.\begin{aligned} r_t &= \mu + \sigma_t z_t \quad \text{with } z_t \sim N(0,1), \\ \sigma_t^2 &= \omega + (\alpha + \phi I_{t-1})(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2. \end{aligned}\right\} \tag{4.2}$$

The parameter ϕ reflects the leverage effect and is usually estimated to be positive. In this case, negative returns increase future volatility by a larger amount than positive returns of the same magnitude. I_{t-1} is 1 if r_{t-1} is negative and 0 if positive.

In Table 3 we show the estimated parameters of a GJR-GARCH with normal noise model as in (4.2). We apply the same methodology to estimate parameters as that described before for calibrating the parameters in (4.1) from the factor strategy returns at constant leverage. We find that all ϕ are null except for currency value. The

TABLE 3 Estimated parameters of a GJR-GARCH model with a normal distribution for the different factor strategies.

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
ω	2.8E-07	3.2E-07	5.9E-07	7.7E-07	2.1E-07	1.3E-07
α (%)	9.8	10.9	8.2	5.7	7.3	6.4
(t-stat of α)	0.7	1.8	0.2	8.2	1.7	0.1
ϕ (%)	0.0	0.0	0.0	5.9	0.0	0.0
(t-stat of ϕ)	0.0	0.0	0.0	5.9	0.0	0.0
β (%)	89.9	88.2	90.0	89.1	91.2	92.9
(t-stat of β)	2.5	8.5	3.1	104.5	1.3	9.9
$\alpha + \beta$ (%)	99.6	99.1	98.2	94.7	98.5	99.3

"Mom" is momentum. The estimation of parameters uses data from January 1, 1990 to December 31, 2013.

leverage effect is thus not present in most factor strategies, although it is an important feature in risky asset classes like equities. Most likely, the long–short portfolios used in the factor strategies neutralize the effect which is observed in the underlying asset class. The foreign-exchange value premium is the one exception.

Another way to impose a relationship between volatility and return is to use a conditional skewness effect. Hansen (1994) and also Campbell and Hentschel (1992) show how to include skewness in GARCH models. The skewed-GARCH model is a GARCH model which follows (4.2) but with the noise z_t replaced by a skewed normal distribution.

In Table 4 on the facing page we show the estimated parameters of a GARCH with skewed normal noise model. As for (4.1) and (4.2), we use maximum likelihood approaches to estimate parameters from the factor strategy returns at a constant leverage. We find a large negative skew effect (ie, skew parameter below 1) for equity momentum, foreign exchange value and momentum and also for government bond momentum, but less pronounced. The skew effect is negligible for government bond value and is even positive for equity value. Momentum factors are thus more likely to benefit from the application of an intertemporal risk-parity strategy due to their negative skew. For equity value, with a positive skew, and for government bond value, with hardly any skew, the strategy may be less efficient at smoothing drawdown events.

TABLE 4 Estimated parameters of a GARCH model with a skewed normal distribution for the different factor strategies.

	Equity		Foreign exchange		Government bonds	
	Mom	Value	Mom	Value	Mom	Value
ω	2.3E-07	2.3E-07	4.4E-07	4.7E-07	6.5E-08	4.8E-08
α (%)	10.1	9.8	7.6	7.4	4.9	5.2
(t-stat of α)	13.2	11.0	13.5	12.4	11.8	12.6
β (%)	89.2	89.6	91.2	91.1	94.8	94.8
(t-stat of β)	129.6	96.3	145.5	127.3	229.7	240.6
$\alpha + \beta$ (%)	99.3	99.4	98.8	98.5	99.7	99.9
Skew	0.88	1.06	0.91	0.88	0.94	0.98
(t-stat of Skew)	60.8	59.6	75.7	72.5	72.8	72.0

"Mom" is momentum. The estimation of parameters uses data from January 1, 1990 to December 31, 2013.

4.4 Intertemporal risk-parity strategies

We have so far looked at the different properties of the volatility of returns of factor strategies at constant leverage. We shall now focus on the impact of these properties on the performance of intertemporal risk-parity strategies applied to factor investing.

The results above (summarized in Table 5 on the next page, obtained from the calibration of different GARCH models, suggest that the application of intertemporal risk-parity strategies to the factors considered should lead to a large improvement of the information ratio of the equity and foreign-exchange factor strategies thanks to volatility clustering and fat-tail events. The benefit should be less pronounced for government-bond factor strategies. The intertemporal risk-parity strategy is expected to smooth maximum drawdown events in equity momentum and in foreign exchange value and momentum thanks to the presence of gain and loss asymmetry. A smaller improvement is expected for government-bond factor strategies and equity value where gain and loss asymmetry is the weakest.

We performed historical simulations of the application of intertemporal risk parity to the factor strategies here considered. We aim to verify the conclusions made above based on the properties of the return and volatility processes observed. We compare the information ratio of the factor strategies at a constant leverage with that at constant *ex ante* risk.

We follow the approach described by Perchet *et al* (2014) and use the I-GARCH model which can be defined from the standard GARCH model propose by Engle and

TABLE 5 Summary of the strength of the parameters reflecting different effects in the GARCH models estimated for the different factor strategies.

Clustering effects

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
Volatility clustering	Strong	Strong	Positive	Positive	Weak	Weak
α	Positive	Positive			Positive	Positive
Probability of extreme events	Weak	Weak	Positive	Positive	Positive	Positive
Student t ν	Positive	Positive				

Gain and loss asymmetry

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
Leverage effect	0	0	0	Weak	0	0
Stronger impact on volatility from negative returns				Positive		
Negative Skew effect	Strong	Negative	Strong	Strong	Positive	Weak
Larger negative return events	Positive		Positive	Positive		Positive

"Mom" is momentum. The estimation of parameters uses data from January 1, 1990 to December 31, 2013.

Bollerslev (1986) by setting the long-term average volatility $\omega = 0$ and $\alpha + \beta = 1$:

$$\left.\begin{aligned} r_t &= \mu + \sigma_t z_t \quad \text{with } z_t \sim N(0,1), \\ \sigma_t^2 &= \alpha(r_{t-1} - \mu)^2 + (1-\alpha)\sigma_{t-1}^2. \end{aligned}\right\} \tag{4.3}$$

This simplification of the GARCH model tends to work well for forecasting volatility in practical terms because the long-term volatility of ω is difficult to estimate and may not even be stationary. As suggested by Hocquard *et al* (2010) we calibrate the model parameters by applying a maximum likelihood approach with an expanding window. The first five years are used for the first parameter estimation and the target volatility used is 5%.

We consider a systematic strategy which invests in a factor strategy changing the leverage C_t of the long–short portfolio in (3.1) in such way that risk of the long–short portfolio is kept at constant *ex ante* target volatility. The leverage is rebalanced on a daily basis.

The volatility of the long–short portfolio needs to be estimated every day. The leverage C_t of the long–short portfolio is simply determined from its current level of *ex ante* volatility σ_t, obtained with all information available at $t-1$, and the predefined target risk budget κ: we observe the performance of the strategy with *ex ante* volatility of the portfolio at the end of day t and with the leverage of the strategy $C_t = \kappa/\sigma_t$ set at the end of the day $t-1$.

The portfolio is rebalanced at close prices and transaction costs were not considered. We shall later discuss the question of turnover and successful approaches to reduce it significantly with only a minor impact on the performance of the strategy.

In Table 6 on the next page we show the results of the historical simulations for the six factor strategies discussed here.

Equity momentum has the largest improvement in its information ratio, which is perhaps not surprising considering that it has two hidden regimes with significantly different volatility and information ratio and the strongest volatility clustering. For foreign exchange value and momentum the improvement is also important as expected from the GARCH parameters and the large difference in volatility and information ratio of the two hidden regimes. For government bond value the benefit of applying a constant risk framework is small, which is in line with our expectations due to the fact that volatility clustering is weak. The benefit of intertemporal risk parity is also not very large for equity value. We could expect a larger benefit when looking at the α parameter of the GARCH model but not only do fat tails and skew point in the opposite direction, the factor also has the smallest difference in the information ratio of the hidden regimes. We observe a good improvement of the information ratio of government bond momentum. The improvement is larger than what we could expect from the α parameter of the GARCH model. The benefit of the strategy comes from an important difference in the information ratio of the two hidden regimes and from fat tails – the estimation of the Student t parameter is quite large for government bond momentum. The maximum drawdown (relative to *ex post* volatility) of equity momentum and government bond momentum is reduced the most. A smaller reduction is found in the other factor strategies as expected from the gain and loss parameters.

In Table 6 on the next page we also find a relatively low significance of the Student t test, at only 60% to 85% for most of the factor strategies. The low significance is mainly due to the short history we used for each factor strategy. The significance should increase with longer history. We just find a much higher significance from the results of the Student t test for equity momentum at about 99% significance levels. The effects are much stronger for this factor strategy than for the others, and thus a shorter period is enough to statistically confirm the benefits of applying an intertemporal risk-parity strategy.

We shall now discuss the impact of the frequency of risk rebalancing in the intertemporal risk-parity strategy. In Table 7 on page 17, we show the results of the historical

TABLE 6 Comparison of constant-leverage factor strategies with the application of intertemporal risk-parity approaches to those same factor strategies.

(a) Constant leverage

	Equity		Foreign exchange		Government bonds	
	Mom	**Value**	**Mom**	**Value**	**Mom**	**Value**
Average annualized excess return (%)	8.4	3.3	1.7	3.9	−0.3	3.1
Average annualized volatility (%)	14.1	9.5	9.1	8.8	5.7	6.1
Information ratio	0.59	0.34	0.19	0.44	−0.06	0.52
Maximum drawdown (MDD)	−63.0	−44.5	−27.8	−34.0	−28.9	−11.4
Ratio MDD/volatility	−4.5	−4.7	−3.0	−3.9	−5.1	−1.9
Average exposure (%)	100.0	100.0	100.0	100.0	100.0	100.0

(b) Intertemporal risk-parity strategy

	Equity		Foreign exchange		Government bonds	
I-GARCH	**Mom**	**Value**	**Mom**	**Value**	**Mom**	**Value**
Average annualized excess return (%)	7.7	2.2	2.5	3.3	0.8	2.9
Average annualized volatility (%)	5.4	5.3	5.3	5.3	5.2	5.2
Information ratio	1.43	0.42	0.46	0.63	0.16	0.57
Maximum drawdown (MDD)	−13.9	−22.1	−14.9	−17.1	−18.1	−9.1
Ratio MDD/volatility	−2.6	−4.2	−2.8	−3.2	−3.5	−1.7
Average exposure (%)	65.5	80.9	69.7	71.7	105.8	99.8
Turnover	7.9	7.8	6.3	4.9	5.3	6.1
Improvement in information ratio	0.83	0.08	0.27	0.19	0.22	0.05
Student test	4.0	0.4	1.3	0.9	1.0	0.2

"Mom" is momentum. Target volatility is set at 5% annualized using forecasted volatility from I-GARCH models. The I-GARCH model parameters are estimated from an expanding window once every year at the start of each year. Results for equity and foreign-exchange factors were obtained from back-tests over the period January 1, 1985 to December 31, 2013 whereas results for government bond factors were obtained over the period January 1, 1988 to December 31, 2013.

simulations with the leverage rebalanced monthly instead of daily to bring the *ex ante* volatility back to target. We found that the improvement in the information ratio is almost the same as with daily rebalancing. But with a monthly frequency the turnover is substantially reduced and so are the transaction costs of implementing the strategy. The most efficient practical implementation of the strategy would involve daily monitoring of *ex ante* volatility and rebalancing only when *ex ante* volatility significantly deviates from target volatility.

TABLE 7 Comparison of constant leverage factor strategies with the application of intertemporal-risk parity approaches to those same factor strategies.

(a) Constant leverage

	Equity		Foreign exchange		Government bonds	
Monthly reblancing of leverage	**Mom**	**Value**	**Mom**	**Value**	**Mom**	**Value**
Average annualized excess return (%)	8.4	3.3	1.7	3.9	−0.3	3.1
Average annualized volatility (%)	14.1	9.5	9.1	8.8	5.7	6.0
Information ratio	0.59	0.34	0.19	0.44	−0.06	0.51
Maximum drawdown (MDD) (%)	−63.0	−44.5	−27.8	−34.0	−28.9	−11.4
Ratio MDD/volatility	−4.5	−4.7	−3.0	−3.9	−5.1	−1.9
Average exposure (%)	100.0	100.0	100.0	100.0	100.0	100.0

(b) Intertemporal risk-parity strategy

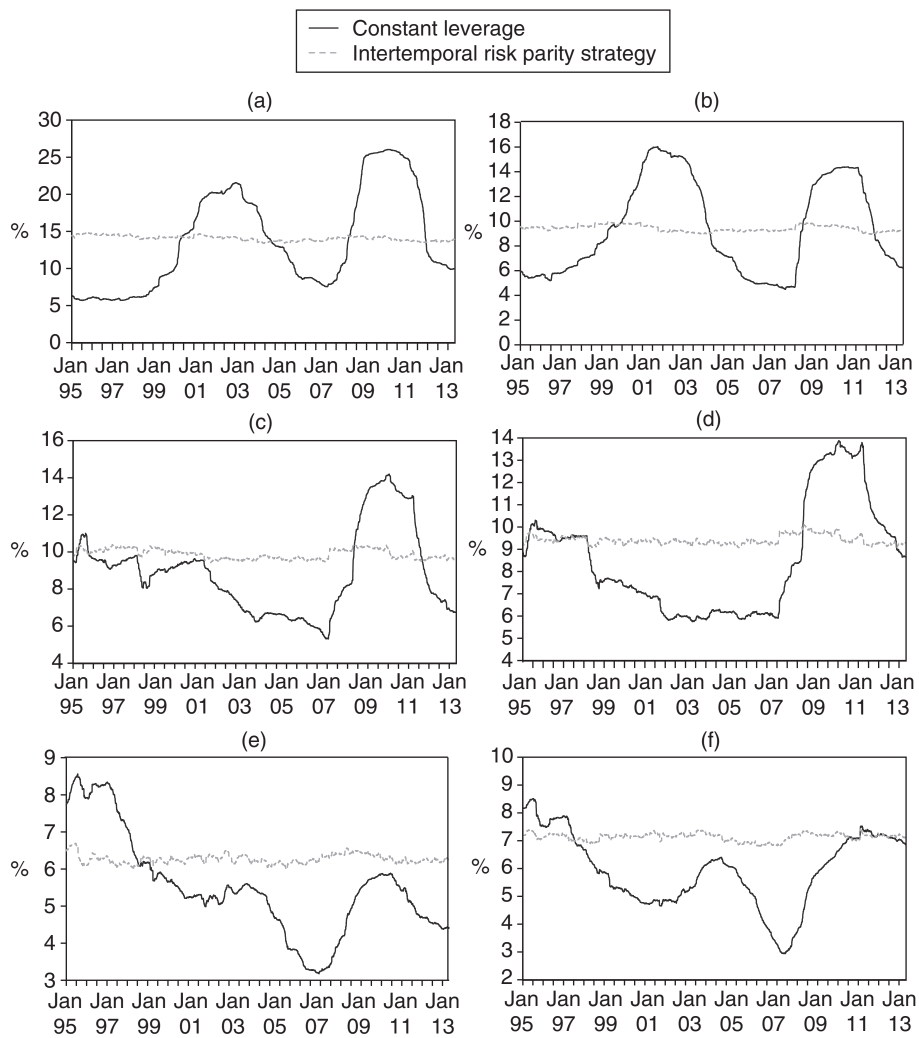
	Equity		Foreign exchange		Government bonds	
I-GARCH	**Mom**	**Value**	**Mom**	**Value**	**Mom**	**Value**
Average annualized excess return (%)	7.6	2.4	2.2	3.3	0.7	3.1
Average annualized volatility (%)	5.7	5.6	5.6	5.5	5.3	5.3
Information ratio	1.33	0.42	0.40	0.60	0.13	0.57
Maximum drawdown (MDD) (%)	−15.3	−22.8	−14.6	−17.5	−18.7	−9.4
Ratio MDD/volatility	−2.7	−4.0	−2.6	−3.2	−3.5	−1.8
Average exposure (%)	66.2	81.4	70.4	72.5	106.7	102.0
Turnover	1.9	1.9	1.6	1.5	1.6	1.6
Improvement in information ratio	0.74	0.08	0.21	0.16	0.19	0.06
Student test	3.6	0.4	1.0	0.8	0.9	0.3

"Mom" is momentum. The difference between these results and those in Table 6 on the facing page is only in the rebalancing of leverage to take into account changes in *ex ante* volatility which is monthly instead of daily.

4.5 The choice of the target volatility κ

An important advantage of intertemporal risk parity over constant leverage is that the *ex post* volatility of returns tends to remain close to target, as shown in Table 6 on the facing page and Table 7, where κ was set to 5%. This is of major importance for asset allocators and portfolio managers who reason in terms of risk budget allocation rather than notional allocation. For them the natural choice of κ is just to set it to the target risk budget they have chosen in their allocation knowing that this risk budget is likely to be respected *ex post*.

FIGURE 1 Volatility of factor returns estimated as the standard deviation of daily returns over the past three years for the factors under the intertemporal risk parity and constant leverage strategies.



(a) Equity momentum, $\kappa = 14.1\%$. (b) Equity value, $\kappa = 9.5\%$. (c) Foreign exchange momentum, $\kappa = 9.1\%$. (d) Foreign exchange value, $\kappa = 8.8\%$. (e) Government bonds momentum, $\kappa = 5.7\%$. (f) Government bonds value, $\kappa = 6.1\%$. The leverage was chosen equal to 1 for all factors. The target volatility of the intertemporal risk parity strategy for each factor was set to the *ex post* volatility of factor returns over the entire period as indicated in Table 1 on page 10 for constant leverage equal to 1.

In Figure 1 on the facing page we show that, even over shorter-term periods, eg, three years, the realized volatility of the intertemporal risk parity strategies remains close to target, whereas the realized volatility of the constant leverage strategies varies significantly. Here the target risk budget κ for each factor is the long-term volatility of the factor at constant leverage equal to 1, as in Table 6 on page 16, eg, for equity momentum κ was set to 14.1%.

It is true that the factor volatility of the factors at constant leverage equal to 1 is not known *ex ante*. Figure 1 on the facing page illustrates well that it is difficult to know *ex ante* what the realized volatility of the constant leverage strategy will be over shorter-term periods, eg, three years. There is, however, a certain level of mean-reversion in the realized volatility of the constant leverage strategies. Thus, we believe that it is reasonable to expect the volatility of factor returns to stay within the range observed in Figure 1 on page 10 in the long-term.

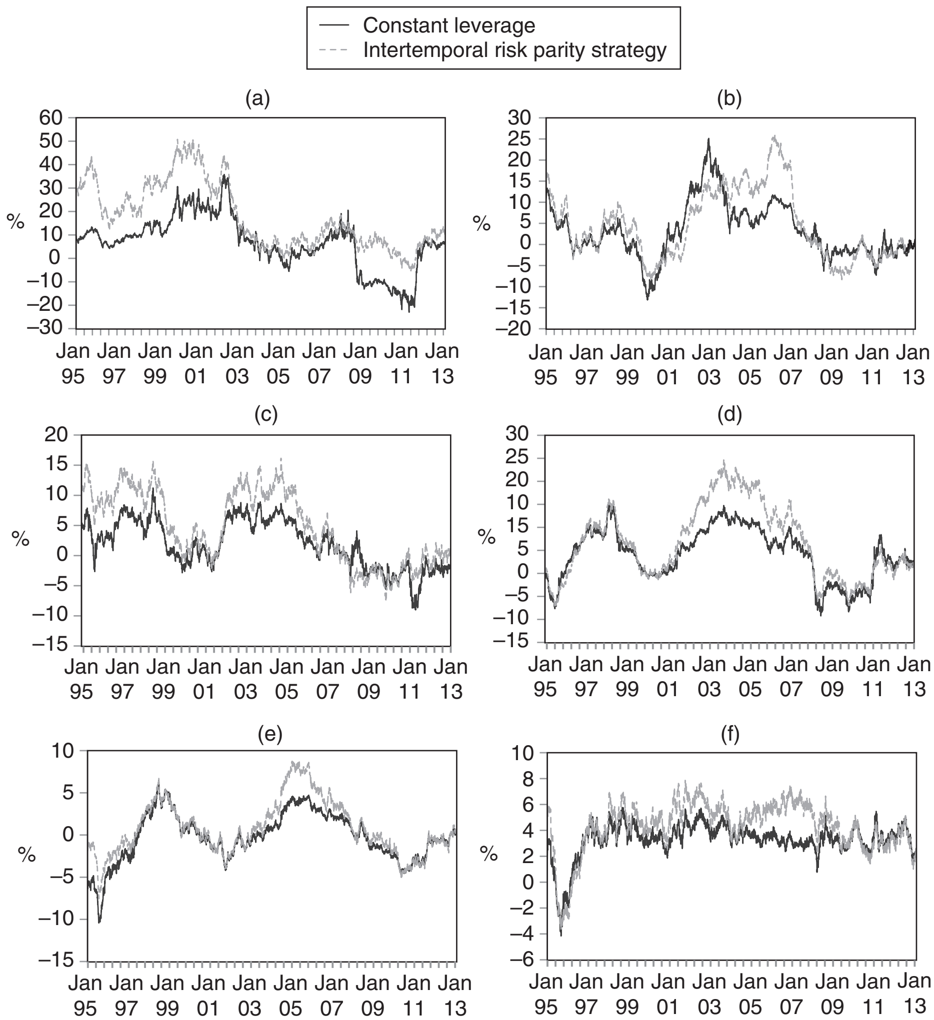
We now ask a simple question: how often in the shorter term, eg, three-year periods, did the constant leverage strategy managed to out-perform the intertemporal risk parity strategy if we set the target risk budget κ as the long-term volatility of the factor at constant leverage equal to 1 as in Table 6 on page 16. In Figure 2 on page 11 we plot the past average three-year returns for both the constant leverage strategies and the intertemporal risk parity strategy with this choice κ. The results show that with the exception of equity value for all other factors the average returns of the intertemporal risk parity are almost all the time either comparable of superior to the average returns of the constant leverage strategy. That means that even from an absolute return point of view the intertemporal risk parity strategy should be considered by investors for as long as the target risk volatility is chosen not too far from the average long-term volatility of the constant leverage strategy.

5 CONCLUSIONS

We confirm that the application of intertemporal risk-parity strategies to factor investing adds value. Adjusting the leverage of the long–short factor portfolio in order to target a constant *ex ante* target risk level adds value when compared with the constant leverage factor strategy. We give evidence of volatility regimes with different average returns in the time series of factor returns and show that the key drivers of the improvement of risk-adjusted returns are volatility clustering and fat-tail events.

The reduction in the largest drawdown is less clear in factor strategies due to the fact that fat-tail events can be observed on both sides of their return distributions. Reduction in the largest drawdown seems to be more pronounced in momentum strategies. Indeed, the skew is more pronounced in the momentum factor strategies while the leverage effect is only present in the foreign exchange value strategy.

FIGURE 2 Average annualized three-year returns of the constant leverage strategy compared to those from an intertemporal risk parity strategy with κ set as the long-term volatility of the constant leverage strategy as given in Table 6 on page 16.



(a) Equity momentum, $\kappa = 14.1\%$. (b) Equity value, $\kappa = 9.5\%$. (c) Foreign exchange momentum, $\kappa = 9.1\%$. (d) Foreign exchange value, $\kappa = 8.8\%$. (e) Government bonds momentum, $\kappa = 5.7\%$. (f) Government bonds value, $\kappa = 6.1\%$.

The factor strategies show that the improvement of the information ratio is also related to the risk dynamics of the underlying asset class. The improvements are much stronger for equities and for foreign exchange than for government bonds, which is in line with the results of Perchet *et al* (2014) for asset classes. They observe a much stronger improvement of risk-adjusted returns in risky asset classes like equities than in low-risk asset classes like government bonds.

In practical terms, we recommend that investors target constant risk exposures to the factors, that they monitor the volatility on a daily basis and that they rebalance the allocation whenever the *ex ante* volatility deviates too much from the target volatility. This should deliver superior risk-adjusted returns with smaller drawdowns than the more traditional constant leverage approaches.

APPENDIX A. GARCH AND MAXIMUM LIKELIHOOD

In finance, GARCH models are used to model the variance of observed returns whenever there is a reason to believe that the variance of past returns will have an impact on the variance of future returns.

$$\left.\begin{aligned} r_t &= \mu + \sigma_t z_t \quad \text{with } z_t \sim t(0,1), \\ \sigma^2 &= \omega + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2. \end{aligned}\right\} \tag{A.1}$$

Let us denote by ε_t the returns residual in (A.1) relative to the mean of the process. The process is thus split into a stochastic piece z and a time-dependent standard deviation σ. The conditional mean of the process is $E(\varepsilon_t \mid F_t) = 0$, where F_t is the filtration defined with the past value of ε_t, of ε_t^2 and of σ^2. The conditional variance is $V(\varepsilon_t \mid F_t) = \sigma^2$. The first two unconditional moments can be computed by iteration with the conditional moments of GARCH model. They can be computed as σ_t^2 is F_t quantified. The unconditional mean process is $E(\varepsilon_t) = E(E(\varepsilon_t \mid F_t)) = 0$. The unconditional variance is defined by iteration, $V(\varepsilon_t) = E(\sigma^2)$. For n iterations and with the condition of $|\alpha + \beta| < 1$, the unconditional variance of the returns is finite and is written as follows:

$$E[\varepsilon_t^2] = \frac{\omega}{1 - (\alpha + \beta)}. \tag{A.2}$$

The variance is positive if ω is the same sign as $1 - (\alpha + \beta)$.

Given a sample $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ of n independent and identically distributed observations which comes from a distribution $f(\varepsilon)$ with unknown parameters θ then the joint density function is:

$$\mathcal{L}(\theta \mid \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) = f(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n \mid \theta) \prod_{i=1}^{n} f(\varepsilon_i \mid \theta). \tag{A.3}$$

The likelihood function changes the point of view, compared with (A.3), by considering the observed values $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ to be fixed parameters of the function, whereas

θ will be the function's variable and allowed to vary freely. The likelihood function is written:

$$\ln \mathcal{L}(\theta \mid \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) = \sum_{i=1}^{n} \ln f(\varepsilon_i \mid \theta). \tag{A.4}$$

In practice, it is often more convenient to work with the logarithm of the likelihood function, called the loglikelihood:

$$\ln \mathcal{L}(\theta \mid \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) = \sum_{i=1}^{n} \ln f(\varepsilon_i \mid \theta). \tag{A.5}$$

An estimate of the value of θ can then be obtained by maximizing (A.5) using numerical optimization.

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