

Decomposing Funding Ratio Risk:
*Providing pension funds with key insights into their liabilities hedge
mismatch and other factor exposures*

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Abstract

In recent years, adverse market conditions demolished the funding status of many defined benefit (DB) pension plans highlighting the need for better risk management. We propose a novel framework to decompose the risk of DB pension plans which differs from earlier work in two fundamental ways. First, while others focused on surplus risk, we give sound reasons to focus on funding ratio risk instead. Second, we include a special *mismatch* factor to measure the sensitivity of the funding ratio to changes in the value of liabilities. We illustrate our framework with a case study based on an actual DB pension fund and decompose its funding ratio risk into mismatch risk and other factor exposures dealing with real interest rates, inflation and economic growth risks.

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JEL codes: C31, C32, D8, G11, G23

Exhibit 1 shows the devastating impact of the 2008-2009 subprime credit crisis and the subsequent euro sovereign debt crisis on the funding ratios of typical defined benefit (DB) pension plans in the US, the UK and the Netherlands. There are differences in starting points and impact sizes but the trends are clear. In just one year, from year-end 2007 to year-end 2008, funding ratios declined rapidly and heavily by about 20% in the UK to 30% in the US and the Netherlands. At their worst since 2007, funding ratios had fallen by an average of about 35% and initial overfunding turned into serious underfunding.

Exhibit 1

Funding ratios recovered but in 2014 remained well below the levels found at the end of 2007. Deteriorating funding ratios and tougher regulations¹, together with a more difficult business climate eroding companies' financial health, put many plan sponsors on track for phasing out or even closing their traditional DB plans. As a result, interest in defined contribution (DC) and/or hybrid pension schemes has been growing rapidly. This emerging trend implies a transfer of risk from employers to employees and retirees. Although this is not the place to comment on the pros and cons of this development, we believe that it is sponsors' fear of being forced to make additional contributions to repair an underfunded status that lies at the heart of such changes. It is in this context and to soften this fear that we want to emphasize the importance of liability-driven investment (LDI) approaches to managing pension funds. An effective LDI approach is inevitably interwoven with effective risk management. It seems to us that pension funds still need to make significant progress in this area. In their recent survey of pension funds and sponsor companies, most of them European pension funds, Badaoui, Deguest, Martellini and Milhau [2014] assessed their views and usage of LDI strategies. One of their main findings was that, although participants were generally familiar with the LDI paradigm, the rate of adoption remained rather limited. In concrete terms, many pension funds were still more concerned with standalone performance than with risk management. For instance, many respondents had not yet translated regulatory minimum funding requirements into strategies to protect the funding ratio by imposing a floor. It is precisely in the area of risk management that this article is making a contribution, by

focusing on funding ratio risk, with the proposal of a quantitative framework to yield insights into the main components of funding ratio risk.

We have two main objectives. The first is to show the advantages of using funding ratio risk over surplus risk. Given existing academic literature, our preference for funding ratio risk as a key risk measure is not necessarily obvious. We shall argue that surplus risk offers a less complete view of the health of the pension fund and at times can even be misleading. In an example, we shall show that an increase in surplus can be accompanied by a fall in the funding ratio, which leaves no doubt about the deterioration of the financial health of the pension fund and tends to be the focus of regulators.

The second objective is to propose a framework for decomposing funding ratio risk. This framework is based on a standard linear factor model. Asset-only applications of such factor models are nowadays frequently used by academics and by practitioners alike, e.g. the Fama-French [1992, 1993] three-factor model or the Carhart [1997] four-factor model for equity portfolios. Similar models are available for major other asset classes as well.² Two reasons warrant that a pension fund needs a tailor-made factor model. First, a pension fund is not an asset-only investor, so customization is needed to account for the fact that its investments are driven by liabilities. Second, pension funds invest not in one asset class but in many. This has repercussions on the choice of which factors to take on board. In particular, a couple of macroeconomic factors, rather than many asset class-specific style factors, might prove to be enough to capture the lion share of the risk exposures.

Funding ratio and surplus

Contrary to what pension fund practitioners and their regulators tend to focus on, most academic references, as well as some practical risk applications, seem to be concerned more with the management of the *surplus*, defined as *assets minus liabilities*, rather than with the management of the *funding ratio*, defined as *assets divided by liabilities*. Because the surplus is linear in its constituent parts, authors may have thought that the linear relationship makes it easier to analyze the statistical consequences of different

investment strategies, unlike a funding ratio approach, which is relative in its constituents.³

There are ample examples in literature of the surplus, or linear, school. Sharpe and Tint [1990] and Leibowitz, Kogelman, and Bader [1992] were among the first to consider the risk-return trade-off with regards to the concept of surplus return. Realizing that the starting surplus itself can have a value of zero or close to zero, implying an undefined or very small or big surplus return, these authors proposed the division of the change in surplus value by the initial value of either the assets or the liabilities. Sharpe and Tint opted for the *assets-centric* definition of surplus return, whereas Leibowitz et al. worked with its *liabilities-centric* form.

In terms of surplus return expectation and variance, there is really not much difference between the two. In order to show this, let A_j , L_j , FR_j , and S_j stand for the values of, respectively, the assets, the liabilities, the funding ratio and the surplus at time j and let the capital R always denote the (discretely compounded) 1-period ahead return where its subscript(s) will clarify which return is meant. In the case of a surplus return, the superscripts ac and lc are used to indicate an *assets-centric* or *liabilities-centric* definition. Given these notational conventions, the *assets-centric* surplus return for period 1 which runs from time 0 to 1 is given by:

$$R_S^{ac} = \frac{S_1 - S_0}{A_0} = \frac{(A_1 - L_1) - (A_0 - L_0)}{A_0} \rightarrow R_S^{ac} = R_A - \frac{1}{FR_0} \cdot R_L \quad (1)$$

and the *liabilities-centric* surplus return is:

$$R_S^{lc} = \frac{S_1 - S_0}{L_0} = \frac{(A_1 - L_1) - (A_0 - L_0)}{L_0} \rightarrow R_S^{lc} = FR_0 R_A - R_L \quad (2)$$

These definitions imply different sizes for the surplus returns, which is obvious (note that $R_S^{lc} = FR_0 \cdot R_S^{ac}$). Crucial is that both definitions imply a linear combination between the random variables R_A and R_L . Both are a kind of adjusted linear excess return of assets over liabilities, where FR_0 adjusts the returns when assets and liabilities are of different size. This linearity opens the way for the mean-variance optimization of the surplus.

Given its analytical tractability, it is not so surprising that many authors worked with surplus returns. Some more recent examples are Scherer [2002], Waring [2004a,b], Coutts and Fleming [2007], Monfort [2008], Waring and Whitney [2009], Ransenberg, Hodges and Hunt [2012] and Ang, Chen and Sundaresan [2013]. Surplus itself has also been used in some practical risk tools. For example, MSCI Barra has recently expanded its BarraOne risk system to include a Beta-module that is able not only to compute the VaR of the surplus but also to break it down into separate risk factor components (see the reference under MSCI Barra [2013]). The same approach is also applied in Legal & General's Prism system [2013a, b], which is based on the PFaroe software of RiskFirst (formerly known as PensionsFirst).

The relative or funding ratio school seems to find fewer advocates in the academic literature. Interestingly, some of the above-mentioned *liabilities-centric* surplus authors published an article in which they went through a mean-variance optimization exercise with respect to the funding ratio return, adding a shortfall constraint (see Leibowitz, Kogelman and Bader [1994]). They assumed that asset and liability returns are log-normally distributed. This implies that the funding ratio return is also log-normally distributed with parameters that can be analytically derived from the underlying log-normal distributions. In this context, a more recent contribution by Swierstra [2011] is relevant. He showed that the funding ratio variable can be approximated via a second order Taylor series expansion, which makes it possible to express the expectation and variance of the funding ratio as non-linear functions of the first two moments of R_A and R_L and of their covariance. Note that, strictly speaking, analytical tractability of the parameters of the distribution of *funding ratio* or *funding ratio return* is a desirable but non-critical assumption. Analytical results can also be obtained via simulations.⁴ Note that Leibowitz et al. [1994] work with the concept of *funding ratio return*, i.e. with $R_{FR} = (FR_I/FR_0)-1$, whereas Swierstra [2011] works with the *funding ratio*, i.e. FR_I , itself. However, from a conceptual and statistical point of view the differences between these concepts are only of minor importance, because the random variables R_{FR} and FR_I are linearly related. In this context still note that R_{FR} can be written as $(1+R_A)/(1+R_L) - 1$, so as a relative and un-weighted excess return of R_A over R_L .

Turning now to some other advocates of the relative or funding ratio school, academics from the EDHEC-Risk Institute developed in a series of related papers an advanced normative framework for optimizing the strategic allocation decisions of pension funds (see Martellini and Milhau [2009], Amenc, Martellini, Milhau and Ziemann [2009] and Deguest, Martellini and Milhau [2013]). They proposed risk factor models in which assets and liabilities are driven by stochastic processes for the equity risk premium, the instantaneous short-rate and the inflation. They showed that an optimal solution formally exists consisting of a liability-hedging portfolio (LHP), a performance-seeking portfolio (PSP) and a dynamic risk overlay that changes the allocation weights through time depending on the evolution of the funding ratio. The objective of the pension fund was expressed in terms of maximizing the expected utility of the funding ratio level at a given long-term horizon of H periods. Relevant regulatory short-term rules for the funding ratio were added as short-term constraints in the optimization model. Martellini and Milhau [2009, p. 21], justified their choice for using the funding ratio rather than surplus as the argument in the utility function:

“This objective, which is perhaps the most natural since it recognizes that what really matters in pension fund management is not the value of the assets per se, but how asset value compares to liability value at each point in time, ...”

and:

“Maximization of expected utility of the funding ratio accounts for the presence of future liability payments since by definition ... it is the asset value net of past liability payment expressed in terms of number of units of the current value of future liability payments.”

The funding ratio is here assumed to be equivalent to the net wealth of the pension fund with total assets expressed in terms of number of units of total liabilities. Brennan and Xia [2002], Van Binsbergen and Brandt [2007] and Hoevenaars [2008] also set as an objective the maximization of the funding ratio.

So far, three main differences between the linear and relative schools have been acknowledged. First, in the industry, the funding ratio is more commonly used than the surplus when discussing the financial health of a pension fund. Second, there are

fundamental differences between the concepts of funding ratio return and surplus return. The former is defined as a *relative and un-weighted* excess return of R_A versus R_L and it is *independent* of the initial funding ratio level FR_0 . In contrast, both forms of surplus return are defined as a *linear and weighted* excess return of R_A versus R_L that is *dependent* on FR_0 , because FR_0 determines the relative importance of R_A and R_L . They are independent of FR_0 only if $FR_0 = 1$. Third, the concept of funding ratio (and funding ratio return) can be directly related to the concept of wealth being expressed in units of liability values. A pension fund with a funding ratio of 2 is, in terms of its liabilities coverage, twice as wealthy as any other pension fund with a funding ratio of 1, irrespective of the absolute size of the liabilities of each fund. Such comparisons are not possible on the basis of surplus alone. At a given point in time, the surplus only distinguishes the size of difference between assets and liabilities, but it does not take the scale of the pension fund's liabilities into account.

Finally, there remains one other difference, which is related to the previous point but which focuses on the evolution of a single fund through time rather than on a cross-sectional comparison of funds at a given moment. Exhibit 2 illustrates this. The example shows that the various definitions of surplus returns for a given pension fund vary between +1.0% to +4.0%. They have one thing in common: all three signal an improvement. In contrast, the funding ratio return signals a deterioration of -1.8%. The two concepts can thus generate fundamentally different signals about the evolution of a pension's funding status.

Exhibit 2

All-in-all, it is clear from the discussion above that the remainder of this paper will focus on funding ratio risk rather than on surplus risk.

RISK METHODOLOGY

Factor Model: assuming an initial funding ratio of 100%

We shall now propose a framework for decomposing the funding ratio risk. The equation below relates the funding ratio at the end of period 1 with the current level of the funding ratio and the next period return of assets and of liabilities:

$$FR_1 = \frac{A_1}{L_1} = \frac{A_0 \cdot (1 + R_A)}{L_0 \cdot (1 + R_L)} \quad \rightarrow \quad FR_1 = FR_0 \cdot \frac{(1 + R_A)}{(1 + R_L)} \quad (3)$$

Let us first focus on the random ratio $(1+R_A)/(1+R_L)$, which drives the stochastics in FR_1 . For simplicity, we shall first assume that the current funding ratio is 1 (or 100%). This assumption will be relaxed in the next section.

We assume that the next period assets return R_A is a linear function of R_L , the next period return of the hypothetical perfect liabilities-hedging portfolio (LHP) - which is thus identical to the next period liabilities return - and other factors. In this section there is no need to be more specific about the other factors. We shall come back to the choice of factors later in the case study. Let's consider the other factors to be given by F_j , with $j = 2, \dots, k$. This means that the assets returns can be written as:

$$R_A = \beta_0 + \beta_1 \cdot R_L + \beta_2 \cdot F_2 + \dots \dots \dots + \beta_k \cdot F_k + e \quad (4)$$

where e is the residual return with an expectation of zero and a homoscedastic standard deviation. The betas are the factor loadings. The first term is the constant β_0 , chosen so that the expectation of the residuals is zero.

By adding 1 to both sides of equation (4) and divide left and right by $(1 + R_L)$ we find that $(1+R_A)/(1+R_L)$ is given by:

$$\begin{aligned} \frac{(1 + R_A)}{(1 + R_L)} &= \frac{1}{(1 + R_L)} + \beta_1 \cdot \frac{R_L}{(1 + R_L)} + \beta_2 \cdot \frac{F_2}{(1 + R_L)} + \dots \\ &\dots \dots + \beta_k \cdot \frac{F_k}{(1 + R_L)} + \frac{(e + \beta_0)}{(1 + R_L)} \end{aligned} \quad (5)$$

By using the fact that $1/(1 + R_L) = 1 - R_L / (1 + R_L)$, we find:

$$\frac{(1 + R_A)}{(1 + R_L)} = 1 + (\beta_1 - 1) \cdot \frac{R_L}{(1 + R_L)} + \beta_2 \cdot \frac{F_2}{(1 + R_L)} + \dots$$

$$\dots + \beta_k \cdot \frac{F_k}{(1 + R_L)} + \frac{(e + \beta_0)}{(1 + R_L)} \quad (6)$$

Letting the * denote the rescaling by $1/(1+R_L)$ and by replacing the left hand side by FR_I (recall that FR_0 was assumed equal to 1), this equation can be simplified into:

$$FR_I = 1 + (\beta_1 - 1) \cdot R_L^* + \beta_2 \cdot F_2^* + \dots + \beta_k \cdot F_k^* + e'^* \quad (7)$$

with $e' = e + \beta_0$. We have five comments about equation (7):

- a) Due to the rescaling by $1/(1+R_L)$, e'^* is an heteroscedastic error term. Therefore, ordinary least squares (OLS) applied to (7) will give inaccurate coefficient estimates.
- b) The factor-loadings $\beta_1, \beta_2, \dots, \beta_k$ can be obtained by OLS to (4), which is in essence the same as applying generalized least squares (GLS) to (7).
- c) Note that $e'^* = (e + \beta_0)^*$ will in general have a mean unequal to β_0 . The mean of e in (5) has a mean of zero and the mean of e' is β_0 . However, due to the rescaling, e'^* in (11) has a mean unequal to β_0 .
- d) e'^* will also be (mildly) correlated with the rescaled factor returns $(R_L^*, F_2^*, \dots, F_k^*)$
- e) $(\beta_I - 1)$ measures, with regards to R_L , the interest rate hedge (IRH) mismatch in R_A assuming that the other factors, F_2 to F_k , are not correlated with R_L . For instance, when R_A loads perfectly on R_L , $\beta_I = 1$ and the coefficient of R_L^* in (7) is 0. This 0 indicates that R_A has no mismatch in tracking R_L . The same reasoning applies when β_I is not equal to 1. For instance, if it is 0.6, one could say that R_A is only tracking R_L on average at 60%. Hence, $\beta_I - 1 = -0.4$ and R_A falls short of tracking R_L by on average 40%. In other words, the IRH mismatch is then -0.4 or -40%. See the Appendix for a formal proof of this interpretation of $(\beta_I - 1)$.

Factor model: the general case

Only one additional step is required when generalizing to the case of an unrestricted funding ratio. In (7) we assumed FR_0 equal to 1. But in fact, since $FR_I = FR_0 \cdot (1 + R_A)^*$, we can now re-write (7) as:

$$FR_0 \cdot (1 + R_A)^* =$$

$$\begin{aligned}
&= FR_0 + (FR_0 \cdot \beta_1 - FR_0) \cdot R_L^* + FR_0 \cdot \beta_2 \cdot F_2^* + \dots + FR_0 \cdot \beta_k \cdot F_k^* + FR_0 \cdot e'^* \\
&= FR_0 + (\beta_1^\# - FR_0) \cdot R_L^* + \beta_2^\# \cdot F_2^* + \dots + \beta_k^\# \cdot F_k^* + (e'^*)^\# \quad \rightarrow
\end{aligned}$$

$$FR_1 = FR_0 + (\beta_1^\# - FR_0) \cdot R_L^* + \beta_2^\# \cdot F_2^* + \dots + \beta_k^\# \cdot F_k^* + e^{general} \quad (8)$$

where $\beta_j^\# = FR_0 \cdot \beta_j$ and $e^{general}$ is the residual term in the *general* model. Equation (8) is the factor model for FR_1 and is the generalization of (7). By multiplying the left and right-hand sides by $(1+R_L)$, followed by a rearrangement of the terms, it is not difficult to demonstrate that (8) implies that:

$$FR_0 \cdot R_A = \beta_0^\# + \beta_1^\# \cdot R_L + \beta_2^\# \cdot F_2 + \dots + \beta_k^\# \cdot F_k + e^\# \quad (9)$$

where now the residual $e^\#$ is equal to FR_0 times e . Thus, the factor loadings in (8) can be obtained by estimating (9), which is the generalization of (4), using OLS. Alternatively, and with equivalent results, one can estimate as before (4) using OLS, followed by multiplying the betas there by FR_0 .

The comments from the previous sub-section also apply here. In particular, d) which here means that the residual $e^{general}$ will be (mildly) correlated with the rescaled factor returns $(R_L^*, F_2^*, \dots, F_k^*)$.

In (8), the general case, the IRH mismatch coefficient, $(\beta_1^\# - FR_0)$, is no longer referenced against 1 but against FR_0 (see again the Appendix). The intuition here is that the only way to get a completely riskless funding ratio is by investing all pension fund assets in a perfect LHP. Suppose a pension fund has a current funding ratio of 1.1, so its assets are 110% of the liabilities. The fund would have to invest 110%, not just 100%, of the liabilities in the perfect LHP. In that case, no matter what happens in the financial markets, R_A and R_L would always be the same and the risk of FR_1 would be zero, because its value would be frozen at FR_0 .⁵ If $\beta_1^\#$ was 1, that would not be enough. There would remain an IRH mismatch of 0.1 leading to IRH mismatch risk. The upside of this risk is of course that the assets value that is not invested in the perfect LHP can be invested

otherwise, which hopefully will lead to positive returns to render the future expected funding ratio larger than FR_0 .

Decomposing funding ratio risk

The classical approach for decomposing the risk of FR_I is variance decomposition. However, Fields [2003] shows that for a linear model such as that in equation (8), the same relative contributions are obtained if one decomposes the standard deviation instead of the variance. For practitioners, the latter is probably more appealing than the variance decomposition. The advantage of volatility, or standard deviation, is that it has the same dimension as the variable under investigation, whereas the dimension of variance would be the square of that dimension.

Because the residual in (8) is correlated with the rescaled factor returns, one must include $e^{general}$ in the decomposition as an explicit factor with a loading of 1. By doing that, the inter-interaction effects between the residual term and each of the other factors are equally divided between the residual and each of the other factors, just as all other interaction effects are treated. So, let the set of relevant rescaled factors be given by $\{R_L^*, F_2^*, \dots, F_k^*, e^{general}\}$ and let F_n^* denote the n^{th} element of this set. Likewise, let their factor loadings in equation (8) be denoted by the set $\{(\beta_1^\# - 1), \dots, \beta_k^\#, \beta_{k+1}^\#\}$ with $\beta_{k+1}^\# = 1$ and let $\beta_n^\#$ denote the n^{th} element of this set. The results from Fields [2003] imply that the volatility of FR_I in (8), defined as $\sigma(FR_I)$, has $k+1$ components that are associated with the $k+1$ rescaled factors. For component $n = 1, \dots, (k+1)$ the following holds:

$$\text{Absolute Volatility Contribution Factor } n = \beta_n^\# \cdot \sigma(F_n^*) \cdot \rho(F_n^*, FR_1) \quad (10)$$

where $\sigma(F_n^*)$ is the volatility of the rescaled factor n and $\rho(F_n^*, FR_1)$ is the correlation coefficient between the rescaled factor n and FR_I . The relative contribution is:

$$\text{Relative Volatility Contribution Factor } n = \frac{\beta_n^\# \cdot \sigma(F_n^*) \cdot \rho(F_n^*, FR_1)}{\sigma(FR_1)} \quad (11)$$

CASE STUDY: SETTINGS

The proposed risk framework, based on a standard linear multi-factor model, is flexible and can be adapted to different settings with different assumptions. Here, we will keep the case study settings as simple as possible.

Forward-looking mind-set

We have made the important choice of using a forward-looking mind-set in our case study, which is based on a typical pension fund portfolio and sheds insight into which risk factors are expected to contribute the most to the overall future funding ratio risk. This mind-set means that the necessary risk decomposition inputs cannot be gathered on the basis of historical time-series data. The difficulty with such an approach is that it would be too rigid in terms of conditioning the analysis on the recent past behavior of financial markets as well as on the current position of the pension fund (its funding ratio and investment strategy). The alternative is to use a forward-looking simulation model which allows for much greater flexibility. Today's interest curves, funding ratio, investment strategy (including, if any, dynamic allocation rules) and projected liability cash-flows, outlooks on the financial markets, etc., can all be used in a forward-looking simulation model.

The simulation model is used to create scenarios describing with equal probability possible future scenarios for the financial asset returns, for the pension fund's balance sheet items and for the factors used in the risk decomposition. These scenarios allow us to compute the needed factor loadings, volatilities and correlations. This forward-looking approach is essentially cross-sectional, i.e. based on simulated data across multiple forward-looking scenarios rather than based on historical time series that all occurred in one single historical scenario.

Simulation model

We used a proprietary forward-looking Monte Carlo simulation model⁶ which consists of two integrated sub-models.

The first sub-model is what we call an economic scenario generator (ESG). Given user-defined inputs regarding start-up values, expectations and uncertainties around the

expectations, the ESG simulates equal probability possible future scenarios for interest curves, inflation, asset class returns and the required risk factors.

The second sub-model is a policy simulator (PS) which comes on top of the ESG. The PS starts with the current data for the pension fund: liability value, assets values, funding ratio, the future liability cash-flows, investment strategy, etc. This data is combined with the ESG outcomes. In the case study, the investment horizon is just one year. For each ESG scenario, the PS computes the pension fund simulated value in one year of its assets, liabilities, funding ratio, LHP, PSP, etc. These numbers form the starting point for further statistical analyses to investigate the expected behavior of a given investment strategy.

The policy simulator is simply a straightforward mechanical model that combines the ESG data with the pension fund data. The real driver of the integrated simulation model is the ESG. We shed some light below on how this works.

In our proprietary euro-based ESG model, three blocks of variables are simulated:

- a) *Economic state variables* like CPI-inflation and various yield curves (nominal and real government bond curves, swap curve and AA corporate bond curve).
- b) *Asset classes/benchmarks* like Euro Government Bonds, High Yield bonds, European Property, Developed Equities World and Commodities. In total 29 benchmarks are covered.⁷
- c) *Specific investment products/instruments* like cash, customized portfolios of bonds with specific maturity constraints and some actively managed products.

Yield curves are simulated via a limited number of factors that describe the full curve. For instance, the real and nominal government curves rest on the Diebold and Li [2006] factorization in which the factors are level, slope and curvature.

Each of the items above-mentioned is simulated using a different simulation model built using a four step approach:

- *Literature scan*: what are the main statistical characteristics (mean, standard deviation, skewness, kurtosis, autocorrelation) of the variables in question and what are its main drivers (correlations with other assets, interest rates, inflation, etc.)
- *Historical data collection*: these data goes as far back as possible so that the simulation equations and their interrelations can be modelled on the basis of long-term time series. For example, the nominal interest curve model and mainstream equity models are based on data spanning at least 40 years.
- *Estimation of a regression equation* that best explains the historical observations including the cross-relations with other variables. In the process of finding the best specification we take into account the insights from the first phase.
- *Examination of the simulation quality* of the equation. If needed, the explanatory variables and/or their coefficients and/or the error term specification and/or the equation specification itself, are recalibrated until we are satisfied with the quality of the equation.

When generating scenario simulations, some initial inputs are required for certain variables, e.g. in the next sub-section we present a swap curve used as initial input for simulating the swap curve in the case study. A number of inputs for the simulations can be chosen so that the simulated expectations and volatilities are in line with the forward-looking assumptions. However, cross-correlations between the simulated variables are not part of those inputs. They are fully implied by the ESG model.

ABC's Model portfolio

The example discussed here is based on an actual Dutch DB pension fund, typical of other similar funds. Its long-term strategic investment plan is expressed in terms of a model portfolio. The exercise was performed shortly after the management team completed its annual review and introduced a number of changes. The new model portfolio was still waiting for formal approval by the board. The management team was seeking a quantitative analysis of the implications of the changes in the allocation on the

funding ratio in the short-term. The funding ratio risk decomposition analysis over one year was thus of particular interest.

ABC follows a top-down investment process that disregards some practical issues, e.g. the active versus passive implementation decisions that are thought to be non-crucial at the strategic level. Asset classes are represented by traditional benchmark indices.

At the top of the allocation pyramid, a distinction is made between the performance-seeking portfolio (PSP) and the liability-hedging portfolio (LHP). A zero-investment interest rate swap overlay is also part of the LHP. It is assumed that the swap portfolio perfectly tracks the relative changes in the liabilities, except for an adjustment for the cost of leverage. The PSP portfolio is constructed on the basis of a longer-term outlook. In theory it should be a well-diversified portfolio that targets a Sharpe ratio as high as possible, even if in practice investors may be satisfied with less. For simplicity, we disregard rebalancing within and between the LHP and PSP over the 1-year simulation period.

Liability outflows as well the contribution inflows to the pension fund by the sponsoring company and its employees are all assumed to take place at an annual frequency. In addition, it is assumed that these flows have just occurred and that they are already reflected in the current values for the assets and liabilities. We further assume that the liabilities are nominal and that the regulator demands that the zero coupon swap curve must be used for determining their present value. In exhibits 3 and 4 we plot the current swap curve and the cash-flows of the liabilities, respectively, used in the case study.

Exhibit 3 and Exhibit 4

Coming back to the LHP and PSP, the pension fund implements a *targeted* interest rate hedge percentage (IRH) via delta hedging on the basis of *money duration* (i.e. modified duration, divided by 100, times invested amount). For instance, if the IRH is 70% then the LHP is constructed so that its money duration is equal to 70% of the money duration of the liabilities. Given the IRH, there are still several possible strategies to implement it. For instance, one could implement it fully with just the swap overlay. All the pension fund assets would then be fully invested in the PSP. In practice pension funds do not go

this far. Some of the assets are used in the LHP, which is then filled with duration-sensitive assets. In ABC's case, only government bonds and corporate bonds are used. Usually, the needed money duration cannot be achieved with these physical LHP assets alone and thus the swap overlay is still required. The PSP can contain interest rate-sensitive assets, e.g. high yield. However, the pension fund's current practice is to fully ignore any interest rate sensitivities of the PSP. The IRH target is thus only implemented via the LHP. This is an important choice and we will come back to it later when discussing the actual risk analysis.

The initial key figures for pension fund ABC are summarized in exhibit 5. Panel A shows the initial balance sheet numbers. Liabilities have a present value of EUR 1,000 million. The assets amount to EUR 1,100 million with over EUR 605 million (55%) allocated to the LHP and EUR 495 million (45%) to the PSP. The initial funding ratio is 110%.

Exhibit 5

Further information on the LHP and PSP is given in panels B and C. The percentage allocations are given in terms of a percentage of the total asset value. The LHP's targeted interest rate hedge is 70%. The liabilities, and the perfect swap, have a modified duration of 19 years. The modified durations of the physical IRH assets are taken from the index providers and amount to 6.9 for government bonds and 6.1 for corporate bonds. The targeted money duration of the LHP should be $0.7 \times (19/100) \times 1,000 = \text{EUR } 133 \text{ million}$. In order to get the EUR 133 million targeted money duration, the swap overlay legs must have notational values of 44.7% of EUR 1,100 million.

The PSP is quite risky, with 39.5% of the 45% allocated to high risk assets (equities, property, emerging bonds, private equity and infrastructure) and only 4% to medium risk assets (high yield and leveraged loans). Finally, there is still a small allocation of 1.5% to cash.

ABC's 1-year ESG forecasts

The model portfolio is a long-term portfolio. For the short-term risk analysis, ABC's management team formulated a number of 1-year forward-looking assumptions for the

ESG. These deal with expectations and standard deviations. The assumptions made are as follows:

- For all curves, the starting point is their current shape. The various interest rate curves are stochastic but expected to stay unchanged over one year. The volatility of changes in the government curve is fully implied by the ESG. The volatilities of swap and credit yield spreads are set in accordance with long-run data observations.
- For the 12-month inflation level, both the expectation and standard deviation are set at 1% for the coming year.
- For the liabilities, the future 1-year returns are fully implied by the given liabilities structure (exhibit 4) and the swap curve simulations. The initial maturities of the cash-flows are all rolled back when the simulations progress in calendar time.
- For the government bonds, the future 1-year return expectations and standard deviations are also fully implied by the curve simulations and the underlying simulation models that translate the nominal government curve simulations into simulated bond-market returns.
- Corporate bonds and each PSP asset are assigned a given 1-year return expectation and standard deviation set consistently with the other assumptions.

Selected factors

As discussed, our first factor, the return on liabilities (R_L) is novel to the existing literature and its role is to measure the risk impact of the interest-rate hedge (IRH) mismatch. For the case study we still need to include the other factors.

There is a large breadth of investment literature on factors. This is not the place to review this literature. For that we refer to two recent overview papers by Koedijk, Slager and Stork [2013, 2014]. It suffices us to make two remarks here. Firstly, a number of papers focus on risk premia factors, e.g. value, size or momentum in equity markets. These are cross-sectional factors that can be used to reweight the market-capitalization benchmark index constituents in smarter ways so as to gain exposure to the risk premia factors and potentially generate higher risk-adjusted returns. Such papers deal with active management and how to efficiently deviate from the market capitalization index. The

resulting strategies are typically packaged as a single factor or multi-factor combinations representing long-short strategies in sub-universes of the benchmark's constituents. By construction, such factors tend to be lowly correlated with the asset-class benchmark itself. Therefore they will not be of much use in our case study, where active management within the asset classes was disregarded. Secondly, we need, first and foremost, factors that are insightful for balanced portfolios. We have already remarked that there is as yet no standard factor model for balanced portfolios. In our case study, we do not want to be dogmatic as to which factors to use or not use. We decided to be pragmatic by following Ransenberg, Hodges and Hunt [2012]. To our knowledge, this is one of the few studies to have considered a factor-based risk decomposition of a pension fund, albeit applied to the pension surplus risk rather than to the funding ratio risk. The authors used four general economic factors capturing 1) real interest rate risk, 2) inflation risk, 3) economic growth risk: credits and 4) economic growth risk: equities.⁸ Unlike us, they did not follow a forward-looking simulation approach. They simply worked with historical time series. Their factors were defined as the net returns on long-short portfolios of asset classes: 1) inflation-linked Treasuries minus 1-month T-bills, 2) nominal Treasuries minus inflation-linked Treasuries, 3) corporate bonds minus nominal Treasuries, and 4) equities minus 1-month T-bills. Given the possibilities within our own ESG simulation model, we took the liberty of pursuing our own route in making the Ransenberg *et al.* factors operational. Where possible, we used true economic factors in terms of interest-rate levels, yield spreads and inflation, rather than asset class returns or excess returns. We will work with six economic factors which are categorized and defined as follows:

Real Rates Risk: The 1-year change (first difference) in the 10-year real zero coupon government bond yield.

Inflation Risk: Two factors used, one forward looking and one backward looking:

- Break Even Inflation (BEI) Risk: the 1-year change in the 10-year BEI rate.
- Actual Inflation Risk: the 1-year change in the actual inflation rate, where inflation itself is measured as the return of the CPI over the last 12 months.

Economic Growth: Credit Risk: Again two factors were considered to distinguish investment grade from high yield credit risk:

- IG Credit Risk: the 1-year change in the spread between the 10-year AA corporate credit yield and the 10-year nominal zero coupon government bond yield.
- HY Credit Risk: the 1-year excess return (discretely compounded rate) of high yield bonds versus investment grade corporate bonds.

Economic Growth: Equity Risk: The 1-year excess return (discretely compounded rate) of an equity composite (80% developed and 20% emerging equities⁹) versus high yield bonds.¹⁰

We chose the 10-year maturity for the various curve-related factors for pragmatic reasons. This lies somewhat in the middle between the durations of the physical hedging assets and the duration of liabilities. Furthermore, 10-year yields are often used by fixed-income investors as a representative benchmark yield.

The various benchmarks underlying the simulations of the last two long-short factors can be found in the footnote of exhibit 5.

CASE STUDY: RISK ANALYSIS

Making the methodology case study-specific

Equation (8) describes in general terms the factor model on which our decomposition approach is resting. For convenience, it can be rewritten as:

$$FR_{1,i}^{ABC} = FR_{0,i}^{ABC} + (\gamma_1 - FR_0^{ABC}) \cdot (R_{L,i}^{ABC})^* + \gamma_2 \cdot F_{2,i}^* + \dots + \gamma_7 \cdot F_{7,i}^* + Unexplained_i^{ABC} \quad (12)$$

where we have now added the subscript i and superscript ABC . The first denotes the observations with respect to simulation scenario i . The latter indicates that the variable in question is ABC-specific. Factors 2 to 7 above are the six factors described in the last subsection. Recall that the “*” denotes a rescaling of the factors by $1/(1+R_L)$. In order to simplify the notation we used γ_j for $\beta_j^\#$ with $j = 1, \dots, 7$ and renamed the rest term.

Equation (9) was the regression equation that must be used to estimate the γ -coefficients. It can now be rewritten as:

$$FR_0^{ABC} \cdot R_{A,i}^{ABC} = \gamma_0 + \gamma_1 \cdot R_{L,i}^{ABC} + \gamma_2 \cdot F_{2,i} + \dots + \gamma_7 \cdot F_{7,i} + Residual_i^{ABC} \quad (13)$$

Recall that in (13) the right hand side variables are now not rescaled. Here $FR_0^{ABC} = 1.1$, the current funding ratio.

All the variables in (12) and (13) were determined on the basis of 10,000 1-year ahead scenarios.¹¹ Therefore, the γ -coefficients in (13) are to be estimated via OLS with $i = 1, \dots, 10,000$. Once the coefficients are known, one computes: a) the observations of the *Unexplained* term in (12), b) the standard deviations of the 1-year ahead rescaled factor observations and of the *Unexplained* term, and c) the correlations of the rescaled factors and the *Unexplained* term with the simulated 1-year ahead funding ratios. Then one uses equations (10) and (11) to make the absolute and relative volatility decompositions.

Model portfolio: expectations and volatilities

The simulation results are presented in exhibit 6 with ABC's 1-year ahead expectations and standard deviations for the main items of the balance sheet and their underlying building blocks.

Exhibit 6

The bottom line, see panel A, is that the funding ratio is expected to improve by 2.72% from an initial 110% to 112.72%, but with a volatility of 7.31%, which indicates a relatively significant downside risk.

In the Netherlands, a crude rule of thumb is that the funding ratio needs at least to be about 125% to 130% to have a sound funding status. At such a level the fund is then likely to be able to start with inflation adjusting its nominal obligations. A funding ratio of 105% is considered by the Dutch regulator as the absolute minimal value for not being underfunded.¹² In this context, pension fund ABC starts in the grey in-between area and is expected to remain there.

Model portfolio: unconstrained volatility decomposition

Panel A in exhibit 7 shows the results without constraining the OLS regression. Column (1) shows the estimated factor loadings in equation (13). T-values are given in column (2). Their sizes themselves are not overly relevant because they are directly impacted by the number of simulation scenarios generated, which at 10,000 is high in our case. The T-values are, however, informative in the relative sense when comparing them with one another. Columns (3) and (4) show the factor volatility of each factor and their correlation with FR_t . Finally columns (5) and (6) show the risk decomposition in the absolute and relative senses.

Exhibit 7

The bottom line is that the factor model explains well the funding ratio risk. Almost 95% of the expected 7.31% funding ratio volatility are explained. Three factors - Equity Risk, IRH Mismatch Risk and HY Credit Risk - stand out with contributions of 20% or more. Three other factors - Real Rates Risk, BEI Risk and IG Credit Risk - have more moderate and negative contributions. Actual Inflation Risk is basically absent.

If we examine panel A further we see that the loading for the IRH mismatch factor is strongly negative. With a targeted IRH of 70%, we would have expected to find a factor loading of about 0.70 minus the current funding ratio of 1.1, i.e. about -0.40. However, we find a value of -0.66. This could suggest that there is something wrong with the way the interest rate hedging was set up. We come back to this in the next sub-section. For the moment we shall focus on the combined role of three factors in panel A. These are the Real Rates Risk, BEI and IG Credit Risk factors. All three deal with 1-year ahead changes in 10-year rates, namely in the 10-year real rate, the 10-year BEI rate and the 10-year credit spread (AA corporate bonds minus nominal government bonds), respectively. If we were simply to sum these changes, we would get exactly the 1-year change in the 10-year AA corporate bond yield. If instead we were to use the factor loadings as weights in the sum we would still get something that comes close to the 10-year credit yield change, because the weight for the 3-factor loadings are approximately 0.4, 0.4 and 0.2, and thus not very far away from equal weights.

The point above is that the three mentioned factors together must closely correlate with the 1-year change in the 10-year swap rate. This is a crucial conclusion. The first factor, the IRH Mismatch Risk, is mainly driven by swap rate changes. Because swap rate changes along the curve are highly correlated, we can conclude that there must be a high (in the absolute sense) correlation between the IRH Mismatch Risk factor on one side and the other three factors on the other. Furthermore, the correlation will be negative because the first factor is a return factor whereas the other three deal with changes in yields or yield spreads.

This all boils down to the conclusion that we have in Panel A a situation of multi-collinearity, which explains not only the highly negative loading on the IRH Mismatch Risk factor but also the negative and quite substantial loadings on the Real Rates, BEI and IG Credit Risk factors. The IRH Mismatch Risk contribution is so high, 40% in the relative contribution, because part of the LHP hedging effect is diversified away by the other three risk factors, which together have a negative relative contribution of -21%. This could suggest that the total IRH Mismatch Risk contribution would be around 19% in total. However, there are other possible explanations contributing to some extent towards this -21% contribution. For instance, the model portfolio's LHP is not perfectly aligned with the liabilities return due to the government and corporate bonds in it. It might well be the case that this misalignment is responsible for some of the -21%.

All in all it is clear that the results in Panel A are biased because of multi-collinearity.

Model portfolio: constrained volatility decomposition

In order to cope with the problem of multi-collinearity, we constrained the OLS regression (13). This can be efficiently achieved using information already available beforehand. For instance, the target IRH is one of the key decisions in the strategy process and, if we assume it is accurately implemented, we could constrain the loading γ_1 in (13) to $0.70 - FR_0 = 0.70 - 1.10 = -0.40$. We can use the simulation data to see how effective the hedge has been. It should be remembered that ABC's management team ignored the PSP when implementing the IRH. It only took the LHP into account. It is possible with a regression analysis to see how large the effective IRH of the LHP has

been in the 10 000 simulated scenarios. The OLS outcomes below contain the answer (T-values between brackets):

$$\Delta LHP_i = 6.36 + 0.66 \times \Delta LIABILITIES_i \quad ,with R^2 = 98\% \quad (14)$$

(80.1) (799.8)

Where, for scenario i , ΔLHP_i and $\Delta LIABILITIES_i$ are changes in millions of euros between time zero and one year in the market value of the LHP and the liabilities, respectively. 0.66 means that changes in the market value of the liabilities are on average hedged for 66% by changes in the market value of the LHP. Here, we will call the 0.66 the “LHP-Centric” effective IRH. Interestingly, with an R-squared of 98%, the two Δ ’s turn out to be almost perfectly correlated. So the delta-hedging process is doing quite a good job despite the small difference: the effective LHP-centric IRH stays slightly behind its target of 0.70. There is a small downward bias in the effective slope coefficient when compared to its targeted size. This may arise from the fact that we ignored convexity effects, that yield changes are not always perfectly parallel along the curve, or that the durations of the physical hedging assets given by index providers may be based on curves other than the swap curve, which should be used here because ABC’s liabilities are valued against this.

Although 0.66 for the LHP-centric effective IRH is compliant with ABC’s IRH implementation process, it is certainly not the most accurate estimate of ABC’s effective IRH. The PSP returns are also likely to be correlated with the liability returns, albeit to a much lesser extent. If one takes these on board as well, one is doing an “Assets-Centric” IRH implementation and the equivalent of regression (14) is:

$$\Delta ASSETS_i = 35.1 + 0.75 \times \Delta LIABILITIES_i \quad ,with R^2 = 53\% \quad (15)$$

(51.0) (105.5)

so the “Assets-Centric” effective IRH is 0.75. Unlike the LHP, the total assets portfolio, i.e. the LHP together with the PSP, is not under-hedging ABC’s target by 4%. Instead, it is over-hedging it by 5%.

The alternative to the unconstrained risk decomposition is thus first to find the effective IRH via a pre-regression of the type (14) or (15). The effective IRH is then added as a constraint to equation (13), which is then estimated via constrained OLS. The rest of the decomposition methodology remains the same as before.

Going back to exhibit 7, in panels B and C we show the factor decomposition based on the constrained procedure. Panel B shows the LHP-Centric decomposition and panel C the Assets-Centric decomposition. Note that the reported loadings for the Mismatch factor are now $0.66 - 1.10 = -0.44$ and $0.75 - 1.10 = -0.35$. These decompositions clearly differ from the unconstrained decomposition in panel A. The differences are well in line with the previous discussion on multi-collinearity. For the Actual Inflation Risk, HY Credit Risk and Equity Risk factors, the differences between the various relative risk contributions are negligible. They all stay well within the 1% range. The relative IRH Mismatch Risk component falls strongly from 40% (Panel A) to 27% (panel B) to 21% (panel C). This is in parallel with similar falls in the diversifying power of the components for the Real Rates factor (from -3.4% to -1.5% to -0.7%), the BEI factor (from -8.3% to -3.2% to -1.0%) and the IG Credit factor (from -7.4% to -2.7% to -0.6%). Finally, it is interesting to note that the Unexplained Risk increases when going from panel A to C. This was to be expected when going from unconstrained to constrained analysis.

The Assets-Centric decomposition is not affected by the biases that popped up in the unconstrained analysis. Furthermore, part of these biases still appeared in the Liabilities-Centric decomposition. Therefore, the Assets-Centric approach is in our view reflecting the most accurate risk decomposition.

IRH Mismatch Risk: a look-through

After having filtered out the precise size of the IRH Mismatch Risk, we can still take the analysis one last further step.

The IRH Mismatch is driven by the liabilities return R_L . In turn, in our case study, the volatility of the latter is fully driven by the swap curve dynamics. This link can be made more explicit. It is not difficult to get a look-through of R_L in terms of swap curve yields.

In fact, using the simulated scenarios it appeared possible to approximate R_L very well with just one single yield maturity point. The best fits were obtained with terms to maturity of about 20 years. Perhaps the most natural maturity point is 19 years, because that point coincides with the initial duration of ABC's liabilities.

Inspired by the relation between the change in a bond's relative price, yield to maturity change, (Macaulay) duration and convexity, we chose a specification that explains R_L from two related variables. Let the 19-year zero coupon swap yield (expressed as a %) at time 0 and 1 year in scenario i be given by $SY_{19,0,i}$ and $SY_{19,1,i}$ and let $\Delta SY_{19,i}$ denote their first difference, then the specification and its estimation results were as follows (T-values between brackets):

$$R_{L,i} = 2.38 - 16.72 \times \frac{\Delta SY_{19,i}}{1 + .01 \times SY_{19,0,i}} + 173.80 \times \left[\frac{\Delta SY_{19,i}}{1 + .01 \times SY_{19,0,i}} \right]^2$$

, with $R^2 = 99.5\%$ (16)

(313.1) (-1316.5) (144.7) .

This equation means that the 19-year swap rate alone is sufficient to explain the full 21.3% relative IRH Mismatch Risk component in panel C in exhibit 7.¹³ Since closely neighboring yields are highly correlated with one another, the same holds true for its close neighbors.¹⁴

When presenting the risk decomposition results such as those in panel C of exhibit 7, it is a matter of taste as to whether or not to look through R_L . If one does, then one only needs to substitute (16), including the estimated coefficients, into the (constrained) factor model (12) and take it from there. That would then also open up the possibility of grouping factors together. For instance, in our case study, before substituting it, one could first rewrite in (16) the swap yield level/change in terms of the real rate, the BEI and the swap spread levels/changes. In the total risk decomposition one could then group together the

two real-rate components and the two BEI components. It is again a matter of taste as to whether one opts for such a regrouping or not.

Finally, pension fund ABC discounts its liabilities with the zero swap curve. Pension funds and/or their sponsors that operate under a different regulatory framework could be required or allowed to discount liabilities differently, e.g. using an AA corporate bond curve or using the expected return on the assets. In such cases one needs to customize equation (16) but the rest still remains the same. In this sense, the risk framework we proposed is universal.

SUMMARY AND FINAL REMARK

Adverse market conditions have demolished the funding status of many DB pension plans in the last decade and showed the need for improved approaches to risk management. In this article we propose a new framework for providing insights into a pension fund's main investment risks. Previous work focused on surplus risk, which in our view is not a useful risk measure – as argued here. Instead, our framework is based on the decomposition of the funding ratio risk, which we believe is the pertinent risk metric for DB pension funds.

To our knowledge, we are the first to propose a decomposition methodology for funding ratio risk. It is based on a factor model and is flexible with regards to the choice of factors. We used a real case study so as to illustrate the application of the framework. The case study was based on forward-looking simulations rather than historical regression, which is in our view more powerful and useful for pension funds. We used Real Rates Risk, Inflation Risk and two economic growth risks, namely Credit Risk and Equity Risk, as risk factors. We also included a risk factor to measure accurately the risk impact of not hedging fully away the interest-rate sensitivity of the liabilities. The case study made clear that our decomposition methodology can give valuable insights about the key risk exposures in the portfolio.

In this article we have used standard deviation, or volatility, as a risk measure. The easiest way to justify this is to assume that the funding ratio distribution is normal. In our case study the normality assumption is reasonable.¹⁵ In cases where a departure from

normal distribution becomes too strong, for instance if non-linear derivatives such as swaptions and/or equity options are used for downside protection of the funding ratio, one could turn to the value at risk (VaR) methodology and decompose the (simulated) VaR at interest.¹⁶

Appendix: Interpretation of the loading on the Liabilities Return Factor when it is Uncorrelated with the PSP Return

If the PSP return is uncorrelated with R_L , then the return on total assets can be written as:

$$R_A = \left(\frac{IRH}{FR_0} \right) \cdot R_L + U \quad (\text{A.1})$$

where IRH is the effective *interest rate hedge ratio* and U is a residual return uncorrelated with R_L . U is driven by the PSP return, any returns in the LHP that are uncorrelated with R_L , and their allocation weights.

The case when $FR_0 = 1$

If the PSP return and R_L are uncorrelated, then equation (6), and so also equation (7), can be written as:

$$\begin{aligned} \frac{(1 + R_A)}{(1 + R_L)} &= \frac{(1 + IRH \cdot R_L + U)}{(1 + R_L)} = 1 + (\beta_1 - 1) \cdot \frac{R_L}{(1 + R_L)} + \beta_2 \cdot \frac{F_2}{(1 + R_L)} + \dots \\ &\dots + \beta_k \cdot \frac{F_k}{(1 + R_L)} + \frac{(e + \beta_0)}{(1 + R_L)} \end{aligned} \quad (\text{A.2})$$

Thus:

$$IRH \cdot R_L + U = \beta_1 \cdot R_L + \beta_2 \cdot F_2 + \dots + \beta_k \cdot F_k + (e + \beta_0) \quad (\text{A.3})$$

This implies that $\beta_1 = IRH$, which in turn implies in (A.2), and thus also in (6) and (7), that:

$$\beta_1 - 1 = IRH - 1 \quad (\text{A.4})$$

which represents the IRH mismatch.

The General Case

The same reasoning can be applied to equation (8). Then (A.4) becomes:

$$\beta_1^\# - FR_0 = IRH - FR_0 \quad (\text{A.5})$$

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Exhibit 1

Funding ratio developments for typical DB pension funds in the US, the UK and the Netherlands^{a/b/c/d}

Country	Funding ratio end 2007	Funding ratio end 2008	Funding ratio Bottom		Funding ratio medio 2014
			since 2007	in Quarter	
USA ^e	113	78 (<i>-31%</i>)	73 (<i>-35%</i>)	Q3 2011	90 (<i>-20%</i>)
UK	110	89 (<i>-19%</i>)	78 (<i>-30%</i>)	Q2 2012	91 (<i>-17%</i>)
Netherlands ^e	142	96 (<i>-32%</i>)	85 (<i>-40%</i>)	Q1 2009	103 (<i>-27%</i>)
mean	122	88 (<i>-28%</i>)	79 (<i>-35%</i>)	/	95 (<i>-22%</i>)

a) USA: based on the "*Pension Fund Fitness Tracker*" covering 500 public companies sponsoring large defined benefit plans; source: UBS Asset Management [2014], Insurancenewsnet.com [2014].

b) UK: based on the "*PPI 7800 index*" covering 6130 (medio 2014) defined benefit schemes; source: Pension Protection Fund [2014].

c) Netherlands: based on the "*Pensioen Thermometer*" (Pension Thermometer) which focuses on the average defined benefit pension fund defined on the basis of around 430 (medio 2014) funds; liabilities are here discounted with the spot swap curve; source: AON Hewitt [2014].

d) Percentual changes since end 2007 are given between brackets and in italics.

e) Approximate numbers (estimated from graphs).

Exhibit 2

Example of conflicting signals of surplus and funding ratio returns

	Assets	Liabilities	Surplus	Funding Ratio
Start value	120.0	90.0	30.00	133.3%
End Value	132	100.8	31.20	131.0%
Returns:				
Normal ^a	10%	12%	4.0%	-1.8%
Assets-Centric ^b	/	/	1.0%	/
Liabilities-Centric ^c	/	/	1.3%	/

a) change in value divided by start value

b) change in surplus divided by start value of assets

c) change in surplus divided by start value of liabilities

Exhibit 3

Initial zero coupon swap curve

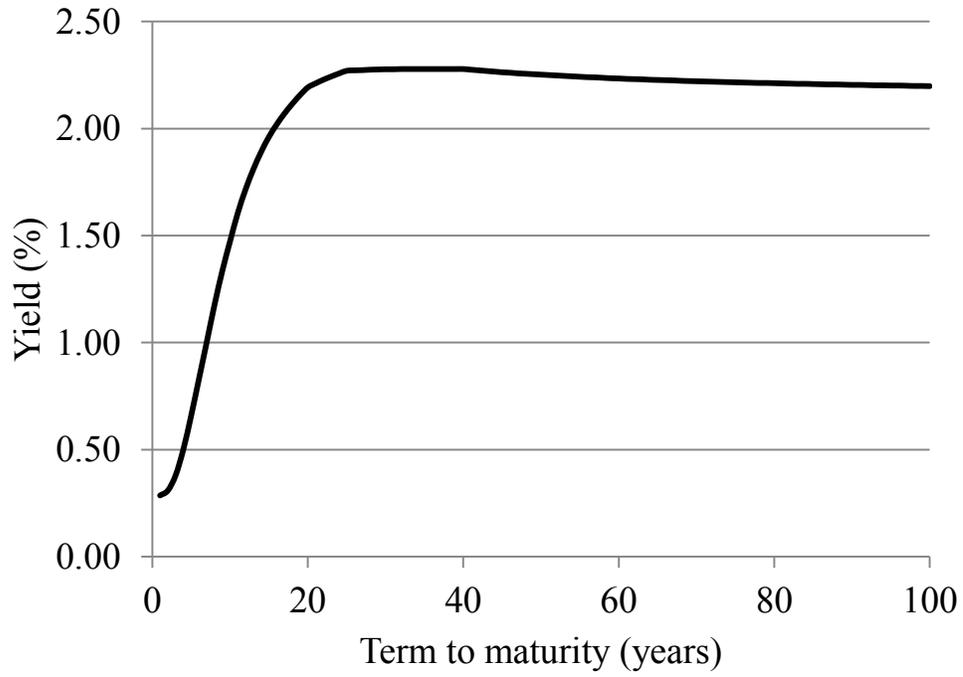


Exhibit 4

Projected outflows liabilities

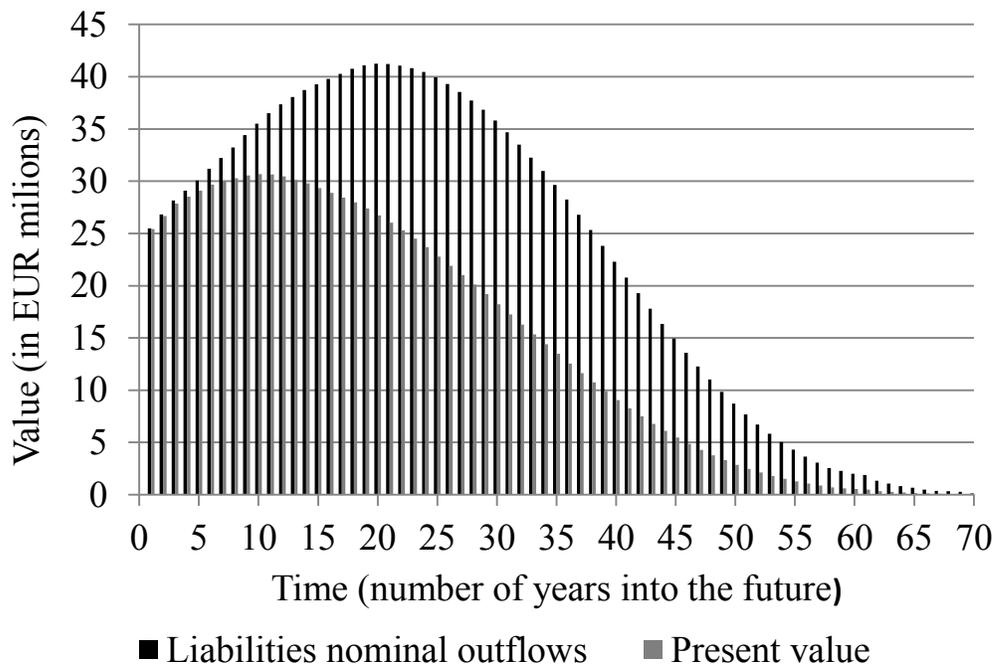


Exhibit 5

Overview Current Balance Sheet and Model Portfolio^a

A) Balance sheet		B) LHP (targeted IRH = 70%)			C) PSP	
Current Value		Allocation	Duration	Money Duration	Allocation	
Assets:		Bonds:			Cash	1.5 %
LHP	605 m EUR	Government	30.0%	6.9	High Yield	1.5 %
PSP	495 m EUR +	Corporate	25.0%	6.10	Leveraged Loans	2.5 %
	1100 m EUR				Bonds_Emerging	4.0 %
		Swap Overlay:			Property	6.0 %
Liabilities	1000 m EUR	long leg	44.7%	19.0	Equities_Developed	20.0 %
		short leg	-44.7%	0	Equities_Emerging	5.0 %
Funding					Private Equity	2.5 %
Ratio	110.0%				Infrastructure	2.0 %
		Total	55.0%	133.0 m EUR	Total	45.0 %

a) The asset classes in panels B and C are defined by the following euro-based references:

- Government = CGBI WGBI All Maturities Index for the Netherlands
- Corporate = CGBI Eurobig Corporates Index
- Cash = 1 month Euribor
- High Yield = Merrill Lynch Global High Yield Index (hedged to euros)
- Leveraged Loans = S&P European Leveraged Loans Index
- Bonds_Emerging = JPM EMBI Global Composite
- Property = GPR 250 PSI Europe Index
- Equities_Developed = MSCI World Index
- Equities_Emerging = MSCI Emerging Markets Index
- Private Equity = Private Equity Index of Cambridge Associates
- Infrastructure = Proprietary Index

Exhibit 6

Model Portfolio: 1-Year Ahead Return Expectations and Standard Deviations

A) Balance sheet			B) LHP			C) PSP		
	Expect.	St.Dev.		Expect.	St.Dev.		Expect.	St.Dev.
Assets:	5.21%	8.66%	Bonds:			Cash	0.29%	0.07%
LHP	4.26%	10.07%	Government	1.85%	3.00%	High Yield	4.30%	10.00%
PSP	6.37%	13.20%	Corporate	2.40%	3.20%	Leveraged Loans	4.00%	7.50%
						Bonds_Emerging	4.60%	13.00%
Liabilities	2.95%	9.19%	Swap Overlay	2.66%	9.20%	Property	5.60%	15.00%
						Equities_Developed	7.00%	17.00%
Funding Ratio:						Equities_Emerging	8.70%	25.00%
Initial Level	110.00%	/				Private Equity	8.60%	16.00%
1-Year Level	112.72%	7.31%				Infrastructure	6.30%	15.00%
Return	2.47%	6.65%						

Exhibit 7

Model Portfolio: Volatility Decompositions for the 1-Year ahead Funding Ratio

A) BASED ON UNCONSTRAINED REGRESSION

	Factor Loading (1)	T-value (2)	Factor Vol (3)	Correlation with FR1 (4)	Absolute Risk (5) = (1)*(3)*(4)	Relative Risk (6)
IRH MisMatch Risk	-0.66	90.00	0.0930	-0.48	0.02928	40.03%
Real Rates Risk	-5.36	-60.74	0.0044	0.10	-0.00247	-3.37%
BEI Risk	-5.19	-58.17	0.0050	0.23	-0.00606	-8.28%
Actual Inflation Risk	0.19	9.12	0.0098	0.04	0.00007	0.09%
IG Credit Risk	-2.84	-49.42	0.0055	0.35	-0.00542	-7.41%
HY Credit Risk	0.46	254.69	0.0865	0.42	0.01690	23.10%
Equity Risk	0.34	386.32	0.1670	0.65	0.03702	50.61%
Unexplained Risk	1.00	/	0.0144	0.26	0.00382	5.22%
Total					0.0731	100.00%

B) BASED ON LHP-CENTRIC CONSTRAINED REGRESSION ASSUMING EFFECTIVE IRH =.66 (66%)

	Factor Loading (1)	T-value (2)	Factor Vol (3)	Correlation with FR1 (4)	Absolute Risk (5) = (1)*(3)*(4)	Relative Risk (6)
IRH MisMatch Risk	-0.44	/	0.0930	-0.48	0.01956	26.75%
Real Rates Risk	-2.41	-37.04	0.0044	0.10	-0.00111	-1.52%
BEI Risk	-2.00	-33.47	0.0050	0.23	-0.00234	-3.20%
Actual Inflation Risk	0.13	5.95	0.0098	0.04	0.00005	0.07%
IG Credit Risk	-1.03	-22.75	0.0055	0.35	-0.00196	-2.67%
HY Credit Risk	0.47	236.87	0.0865	0.42	0.01717	23.48%
Equity Risk	0.34	349.81	0.1670	0.65	0.03663	50.08%
Unexplained Risk	1.00	/	0.0160	0.32	0.00513	7.02%
Total					0.0731	100.00%

C) BASED ON ASSETS-CENTRIC CONSTRAINED REGRESSION ASSUMING EFFECTIVE IRH =.75 (75%)

	Factor Loading (1)	T-value (2)	Factor Vol (3)	Correlation with FR1 (4)	Absolute Risk (5) = (1)*(3)*(4)	Relative Risk (6)
IRH MisMatch Risk	-0.35	/	0.0930	-0.48	0.01538	21.03%
Real Rates Risk	-1.14	-16.13	0.0044	0.10	-0.00052	-0.72%
BEI Risk	-0.63	-9.75	0.0050	0.23	-0.00074	-1.01%
Actual Inflation Risk	0.11	4.54	0.0098	0.04	0.00004	0.06%
IG Credit Risk	-0.24	-4.99	0.0055	0.35	-0.00047	-0.64%
HY Credit Risk	0.47	219.82	0.0865	0.42	0.01729	23.64%
Equity Risk	0.34	320.93	0.1670	0.65	0.03646	49.84%
Unexplained Risk	1.00	/	0.0174	0.33	0.00570	7.79%
Total					0.0731	100.00%

NOTES

¹ For example, new amendments to IAS 19 became effective in January 2013. These implied stricter rules for (listed) companies implying that both a pension fund's assets and liabilities have to be valued marked-to-market. This leads to a greater volatility on the sponsor's balance sheet of the net pension position. Another example is that in the Netherlands stricter regulation was introduced in January 2015 to reduce the probability of underfunding. The Dutch government also started in 2014 an initiative to examine possible structural changes to make the pension system more future proof.

² For example, Fama and French [1993] proposed a two-factor model for bonds, and Fung and Hsieh [2004] developed a seven-factor model for hedge funds. Recently, Della Corte, Riddiough and Sarno [2014] suggested a two-factor model for currencies. There is no generally accepted factor model for balanced portfolios.

³ The use of the funding ratio or the surplus does not necessarily rule each other out. Leibowitz et al. [1994], for instance, optimize the funding ratio in a (one-period) risk-expected return sense with a constraint on shortfall risk that is formulated in terms of funding ratio return. One can, however, alternatively rewrite such a constraint as a constraint on the surplus. This can also be turned around. A surplus constraint, for instance the constraint that the surplus may not decline (possibly with the eye on reporting considerations for sponsor's balance sheet) can thus alternatively be expressed as a constraint on the funding ratio.

⁴ Leibowitz et al. [1994] and Swierstra [2011] overlooked a potential issue when assets and liabilities are both normally distributed. In such cases, their ratio follows the Cauchy distribution which has undefined moments. However, as long as the outcomes in the numerator are not approaching zero, this issue is not likely to cause a concern. In our practical application with funding ratios, this issue does not seem to be of major concern.

⁵ For simplicity, we assume that the amounts of future liabilities outflows do not change between the current level and the end of period 1. However, their present values will change because their maturities decrease and their discount curve changes as well.

⁶ The simulation model is developed and maintained by the Financial Engineering team at XXXXXXXXXXXX and is used by various portfolio management teams for strategic analyses, in particular in the areas of retirement and solvency II solutions.

⁷ These can be broken down into: government bonds 5x, credits 3x, hybrids 2x (convertible bonds and emerging markets debt), property 3x, equity 9x and alternatives 7x. Of these there are still 13 benchmarks that are available in two currency hedging versions, namely unhedged and fully hedged.

⁸ For simplicity we disregard here another factor that Ransenberg et al. [2012] used to account for actuarial uncertainties regarding future liability cash-flows.

⁹ For simplicity we assumed that there was no rebalancing during the year.

¹⁰ All our factor definitions are incremental to their preceding factors. Ransenberg et al. [2012] uses a similar approach except for equity risk. It is not clear to us why they did not also define equity risk in an incremental way.

¹¹ In this article, returns are expressed as discretely compounded rates of return.

¹² The 105% still takes a safety cushion into account for unforeseen expenses and risks.

¹³ The second independent variable in (16) is capturing convexity effects. It has not much incremental explanatory power. The first variable alone, measuring duration effects, has already a R-squared of 98.46%.

¹⁴ For instance, we estimated (16) for maturities of one to 30 years. The maturities from 17 to 29 years all had an R-squared above 99%. The highest was 99.85% at 22 years. At the shorter end of the maturity spectrum the R-squareds are lower but rise quickly: 38% at one year, 81% at five years and already 95% at 10 years.

¹⁵ The estimated skewness and excess-kurtosis coefficients for ABC's model portfolio are only .50 and .41, respectively.

¹⁶ See Hallerbach's [2002/2003] discussion of the VaR decomposition in a simulation context. However, the precise way to apply this in our approach is still subject of further research.