Inter-temporal risk parity: A constant volatility framework for equities and other asset classes

In a world where asset returns follow a normal distribution pattern, the volatility of those returns is constant over time. Hence, in such a world, allocating weights to assets in a portfolio is equivalent to determining a risk budget; they are two sides of the same coin. However, the real world is more complex. The returns of financial assets do not follow normal distributions and volatility is not directly observable. In this summary of our most recent white paper, Romain Perchet of the Financial Engineering team explains why this is, and how targeting a constant risk budget over time improves the risk-return trade off in a number of key asset classes.

1 This is a summary of a white paper written by Romain Perchet, Raul Leote de Carvalho, Thomas Heckel and Pierre Moulin. The white paper in its entirety is available by clicking on the link above or by download from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2384583
There is evidence that managing equities to target constant volatility adds value compared to buy-and-hold strategies. Managing equities to target constant volatility means buying more equities when volatility is low and selling equities and investing in cash when volatility is high, in such a way that portfolio volatility remains constant; in other words, targeting inter-temporal risk parity.

Inter-temporal risk parity is a systematic strategy that invests in both a risky asset and the risk-free asset, rebalancing the portfolio in such way that the portfolio risk is kept at a constant pre-defined target level. The weight of the risky asset in the portfolio is always positive and can, if necessary, be leveraged. The weight of the risk-free asset can be positive or negative, depending on whether the risky asset must be deleveraged or leveraged so as to attain the constant target risk. To explain the benefit of this strategy, we used Monte Carlo simulations based on a number of time-series parametric models from different volatility models, which allowed us to model a number of effects.

In our first simulation, we simply demonstrated that in a world where the returns of risky assets are normally distributed and volatility is constant over time, the application of the strategy using a rolling historical standard deviation of returns as a model for volatility neither adds nor destroys value. It simply generates a higher average exposure to the risky assets, which explains the higher excess return and greater volatility of the strategy. Ultimately however the final Sharpe ratio of the strategy is comparable to that of a buy-and-hold strategy.

The fact that the volatility of financial asset returns tends to show positive auto-correlation over several days is known as volatility clustering, meaning that high-volatility events tend to cluster over time. When volatility clustering is introduced in the time series of risky asset returns, we find in simulations that the inter-temporal risk parity strategy delivers a higher Sharpe ratio than a buy-and-hold strategy does. The volatility clustering effect is in essence a market timing effect. If the volatility changes over time and returns remain constant, then the Sharpe ratio is higher in lower volatility regimes. Increasing the weight of the risky assets in such periods will result in better risk-adjusted performances.

The inter-temporal risk parity strategy can add even more value when the returns of the risky assets show not only volatility clustering but also include fat tails. Then, not only do we find a larger improvement in the Sharpe ratio, we also see a clear reduction in the largest drawdowns. Compared to buy-and-hold, we find higher returns and lower volatility. The strategy requires leveraging the risky asset in low volatility regimes but still achieves lower volatility than a buy-and-hold strategy.

![Improvement of Sharpe ratio according to clustering effects and fat tail effects](chart)

**Improvement of Sharpe ratio according to clustering effects and fat tail effects**

Impact of the clustering effect and fat tails in the Sharpe ratio of an inter-temporal risk parity strategy. Upper chart: clustering impact changes from 60% (high effect) to 2% (small effect). Bottom chart: fat tails events change from 3 (frequent fat tails events) to 30 (low frequency of fat tails events). The clustering impact, at 7%, is in line with historical estimates for the S&P 500. In both charts, volatility = 18.8%, the target risk budget is chosen so as to target 18.8% of volatility for the risky assets, the target Sharpe ratio = 0.40 and 500 Monte Carlo simulations of 2 600 daily returns were used.

BNP Paribas Investment Partners, January 2014

If instead of returns being are not constant over time we observe a negative correlation between volatility and returns, then an even stronger improvement in the Sharpe ratio should be expected when compared to a buy-and-hold strategy. For equities and for high yield bonds, this is typically the case.

We also looked at the impact of the rebalancing frequency. The results of the simulations at different frequencies show that if the frequency of rebalancing is lowered then the improvement in the Sharpe ratio becomes less significant. However, the results for equities change little if the frequency is reduced from daily to weekly. This is important because it means that the turnover of the strategy can actually be largely reduced in practical applications.

Perfect volatility smoothing would require perfect volatility foresight, which is impossible as volatility is not even observable. The most popular approach to forecasting volatility is to use time-series models. One reason for this is the fact that these models require only easily accessible historical information. To use such models, we must first define the features required when modelling volatility and then put these into an actual financial distribution. If successful, it is reasonable to expect that the final model should be successful in forecasting volatility. The key expected features are volatility...
clustering and volatility asymmetry. We focused our attention on four such models in the family of GARCH models.

We performed simulations to assess which of the four chosen GARCH models generates the superior volatility forecast when applied to the S&P 500. To assess this, we looked at the ex-post volatility of the inter-temporal risk parity strategy using daily data within a one-year rolling window and checked which of these models produces the best control of volatility ex-post. In the historical backtest, the I-GARCH model clearly demonstrates superior control of the volatility of the portfolio. The one-year rolling ex-post volatility deviates less from the target volatility of 10% when the I-GARCH model is used. With other models, the ex-post volatility falls well below the target between 1991 and 1996 and again between 2003 and 2008. This fall in volatility is mainly due to the long-term volatility parameter, which is the most difficult to estimate.

I-GARCH best model to target volatility over time

Comparison of the one-year rolling ex-post volatility of an inter-temporal risk parity strategy applied to the S&P 500. The target volatility is 10% and the forecast volatility is based on four different GARCH models, with parameters estimated from an expanding window once every year at the start of each year.

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We also looked at the performance of the strategy over this historical period. When compared to buying and holding the S&P 500, the inter-temporal risk parity strategy improves the Sharpe ratio and reduces the maximum drawdown, irrespective of the GARCH model used. We can say that the S&P 500 shows volatility clustering and we observed some short-term serial correlation in the returns.

We recommend the use of I-GARCH models in practical implementations of the strategy. This model for volatility shows the strongest predictive power and manages to keep ex-post volatility reasonably close to target. The improvement in the Sharpe ratio and reduction in drawdowns when compared to buy-and-hold strategies was better than that found using other GARCH models.

Finally, we looked at the application of the inter-temporal risk parity strategy against other equity indices and in other asset classes. Assets with stronger volatility clustering and fat tails, e.g. high yield bonds, show, as might be expected, the most significant improvements. These effects are also significant for equities but less so for commodities. For investment-grade corporate bonds and government bonds, volatility clustering has not been strong enough in the last 20 years to generate any significant or visible effects.

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<tr>
<td>Average annualized excess return</td>
<td>7.1%</td>
<td>5.7%</td>
<td>6.1%</td>
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<tr>
<td>Average annualized volatility</td>
<td>18.8%</td>
<td>9.5%</td>
<td>9.9%</td>
<td>10.4%</td>
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<td>Sharpe ratio</td>
<td>0.39</td>
<td>0.45</td>
<td>0.44</td>
<td>0.40</td>
<td>0.49</td>
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<tr>
<td>Maximum drawdown (MDR)</td>
<td>-52.5%</td>
<td>-30.0%</td>
<td>-30.0%</td>
<td>-28.6%</td>
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<td>Average exposure</td>
<td>1.00%</td>
<td>0.92%</td>
<td>0.93%</td>
<td>0.94%</td>
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<td>Improvement in Sharpe ratio</td>
<td>0.06</td>
<td>0.04</td>
<td>0.11</td>
<td>0.06</td>
<td>0.07</td>
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Comparison of the one-year rolling ex-post volatility of an inter-temporal risk parity strategy for different asset classes with an inter-temporal risk parity strategy, with target volatility at 5% and using forecast volatility from I-GARCH models. The GARCH model parameters are estimated from an expanding window once every year at the start of each year.

Bloomberg, BNP Paribas Investment Partners, January 2014

This paper highlights the importance of risk management in asset allocation portfolios. The fact that volatility is not constant over time and tends to exhibit volatility clustering, which makes it easier to forecast, is of great importance. Investors should think in terms of risk budget allocation rather than fixed weights. Inter-temporal risk parity strategies dynamically adapt the asset class weights so as to target a desired risk budget quite successfully, and can show better risk-adjusted returns. Moreover, since risky asset classes also show fat tails, inter-temporal risk parity strategies can smooth their impact and reduce drawdowns relative to buy-and-hold strategies that rebalance. The fact that, in some asset classes, returns are on average lower in periods of higher volatility brings additional benefits, further improving the Sharpe ratio for those asset classes when targeting a constant risk. For less risky asset classes such as government bonds, the strategy shows little added value other than keeping the risk budget constant. This however may be a consequence of the period used in our simulations, which was extremely benign for government bonds.

Written 3 February 2014

Click here to read full white paper
The framework developed by BNPP IP, briefly described in this article, is more comprehensively presented in the white paper “Determining a strategic asset allocation in a Solvency II framework”. If you are interested, please contact Sophie Debehogne, Investment Specialist in charge of customised and LDI solutions (sophie.debehogne@bnpparibas.com).

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