Towards Second Generation Equity Risk-Based Strategies

Raul Leote de Carvalho

is head of Quantitative Strategies and Research in the Financial Engineering team at BNP Paribas Investment Partners in Paris, France.

raul.leotedecarvalho@bnpparibas.com, Tel. +33 (0)1 58 97 21 83

Xiao Lu

is a quantitative analyst in the Financial Engineering team at BNP Paribas Investment Partners in Paris, France.

xiao.lu@bnpparibas.com, Tel. +33 (0)1 58 97 75 64

Pierre Moulin

is head of Financial Engineering at BNP Paribas Investment Partners in Paris, France.

pierre.moulin@bnpparibas.com, Tel. +33 (0)1 58 97 20 52

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BNP Paribas Asset Management, 14 rue Bergère 75009 Paris, France

The Market Capitalization Portfolio and Market Inefficiency

The strong-form of the Efficient Market Hypothesis[1] (EMH) assumes that financial markets are efficient in the sense that the stock prices reflect all available information at any point in time. A consequence is that no investor should be able to consistently achieve returns in excess of the average market returns, on a risk-adjusted basis, given the information available at the time the investment is made.

The EMH has been historically linked to random-walk models. The first model of stock prices dates back to 1863 when the French broker Jules Regnault. Louis Bachelier, a French mathematician, in his 1900 PhD thesis, "The Theory of Speculation" revisited this model. The work of these two pioneers was ignored until in the 30s and 40s when Alfred Cowles suggested that even professional investors appeared unable to out-perform the market. But it was only in the 60s that the EMH, coined by Eugene Fama in his PhD thesis, gained momentum as an academic concept.

The EMH underpins the Capital Asset Pricing Model (CAPM) which was developed independently by Jack Treynor (1962), William F. Sharpe (1964), John Lintner (1965) and Jan Mossin (1966). The CAPM model assumes that the risk of a stock or portfolio can be separated into systematic risk and diversifiable risk, also known as specific risk or idiosyncratic risk. Diversifiable risk is specific to each stock, i.e. not correlated with other risks, and can therefore be diversified away by adding more stocks to a portfolio. The market capitalization portfolio is the most diversified portfolio and has no exposure to stock specific risk: the exposure to specific risk is fully diversified away. Systematic risk cannot be diversified away and is represented by a quantity called beta, which measures the exposure of a given stock or portfolio of stocks to the market capitalization portfolio. Beta is additive so the beta of a portfolio is the weighted average of the stock betas.

Beta is the product of two factors. The first factor is the correlation of a stock or portfolio returns with the returns of the market capitalization portfolio. The second is the ratio of the volatility of the stock or portfolio to the volatility of the market capitalization index.

In CAPM, stock and portfolio returns are uniquely determined by their beta, i.e. their exposure to the market capitalization portfolio. The model assumes that there is one common

factor, the market, which explains all correlations of stock returns, captured entirely by the beta of each stock. All returns in excess of beta, the stock specific returns, are by consequence uncorrelated from one stock to another and have an expected zero mean-return. The CAPM model is therefore consistent with the EMH and establishes that after costs it is impossible to outperform on a risk-adjusted basis the market capitalization. Indeed, in CAPM the exposures to stock specific risk generate random specific returns and thus cannot be predicted. The consequence is that the market capitalization portfolio, being the most diversified portfolio with no exposure to specific risk, must have the largest possible risk-adjusted return, which gave it the particularly important status of reference or benchmark for any investment in equities.

The EMH and CAPM catapulted the market capitalization portfolio into the position of natural benchmark for any equity investment. Moreover, the market capitalization involves hardly any trading: it is the most scalable and liquid strategy to investing in equity markets, and will always remain so. Any deviations from the market capitalization portfolio will necessarily involve higher turnover, higher transaction costs and also market impact, a function of assets under-management, which will always detract from portfolio returns of any active strategy.

Yet academics and fund managers have challenged the validity of CAPM ever since its introduction. Empirical tests show that stock correlations cannot be fully described by just one factor as CAPM assumes. The idea that stock specific returns, the stock returns in excess of stock beta, may exhibit some level of predictability and may not be just random noise gained credibility in 1992 when Eugene Fama himself and Kenneth French showed that portfolios tilted towards either value stocks or small capitalisation stocks do outperform the Market Capitalisation Portfolio (MCP) over the long term. These have been observed since and are known as pricing anomalies. These two pricing anomalies known as value and small-cap anomalies are clear violations of EMH and therefore examples of market inefficiencies. The question has always been whether they will eventually disappear proving that, after all, EMH is alive. But empirical evidence that these anomalies can persist for decades suggests otherwise.

Other pricing anomalies or inefficiencies are known. Stock momentum for example, first presented by Jegadeesh and Titman (1993, 1999) gained acceptance following the work of

Carhart (1997): portfolios tilted towards stocks with the highest returns in the preceding months outperform the market capitalization portfolio in future months. The momentum anomaly is the tendency that rising stock prices tend to extend the rise further and conversely falling stock prices tend to continue to fall further. Indeed, the stocks with the best performances in the previous 12 months do tend to extend their out-performance and the stocks with the poorest performances do tend to continue to under-perform.

But tilts in favour of value stocks, small-caps or momentum stocks only create portfolio strategies that outperform the market capitalization portfolio when trading costs and market frictions are kept low. Value, Size and Momentum belong in the world of active management. Unlike the market capitalization portfolio, which involves hardly any trading and offers the largest liquidity, the latter require higher turnover, involve active choices of indicators and rebalancing strategies which impact returns. Their performances also depend strongly on the assets under management. The larger the assets committed to these active strategies, the more market impact will detract from average portfolio returns.

Active management thrives on the idea that CAPM is not observed empirically and that some active managers can out-perform the market capitalization index. Nevertheless, the game of out-performing the market capitalization portfolio will remain a zero-sum game and for some investors to out-perform, some other investors must under-perform when investing in portfolios tilted away from the market capitalization portfolio in the opposite direction.

Behavioural finance and market inefficiencies

Behavioural economists attribute market inefficiencies to *irrational* investor behaviour like overconfidence, overreaction, information bias and other cognitive biases. These are due to the fact that people do tend to make predictable errors in reasoning and processing information.

The value anomaly is explained by behaviouralists as arising from the fact that most investors over-estimate earnings growth in growth stocks and end up paying too much for those stocks. The result is that value stocks are cheap and tend to have higher returns than predicted by the CAPM-beta whereas growth stocks have lower returns than predicted by the beta. Investors aware of this generalized irrationality can profit from the anomaly by investing in bargain value stocks and avoiding the over-reaction selling of growth stocks.

The small-cap anomaly, also known as the neglected firm anomaly, is explained by behaviouralists by the fact that these companies are less covered by analysts and therefore less known. Information available for these companies is, in many cases, restricted to what is strictly required by law. Abnormally higher returns are likely generated by the less interest in these companies and also for liquidity reasons. Large investors, in particular, tend to shun them for liquidity reasons. Even index-managers replicating the market capitalization portfolio often do not invest in them simply because when these stocks are very small they also have very little impact on the tracking error risk and the excess return of the index replicating funds they manage.

Behaviouralists explain the momentum anomaly by the irrational under-reaction of investors to new information, failing to incorporate it in stock prices. In particular, investors seem to find it difficult to sell stocks which generate losses in their portfolios despite the arrival of new information that could push them into doing so and investors also tend to jump on the bandwagon of a positive trend even when information may suggest that these stocks are already expensive.

The low risk anomaly in the equity markets

The low risk anomaly is yet another inefficiency for which there is a large breadth of empirical evidence. It simply translates the fact that portfolios invested in low risk stocks have produced historically much higher risk-adjusted returns than predicted by their CAPM-beta and that, conversely, portfolios invested in the riskier stocks have produced much lower risk-adjusted returns than predicted by their CAPM-beta.

The anomaly is known since the 1970s and was first reported by Robert Haugen and James Stein (1972) in a working paper entitled “On the Evidence Supporting the Existence of Risk Premiums in the Capital Market”. Haugen and Stein focussed on volatility as the measure of risk and showed empirically that portfolios invested in the least volatile US stocks had systematically produced the highest risk-adjusted returns and that portfolios invested in the riskier stocks had produced the lowest risk-adjusted returns. They used the CRSP database and their analysis covered the period of 1926 through 1969. The anomaly is in fact not only observed in the US market but practically everywhere as demonstrated recently by Nardin Baker and Robert Haugen (2012).

The low risk anomaly is simple to demonstrate empirically. We have taken the stocks in the MSCI World Index[2] from the Nov-95 to Sep-09. At the start of each month we estimated the historical volatility of each stock from the standard deviation of the previous three years of weekly returns. Stocks are then ranked by their historical volatility. At the start of each month we form ten portfolios, each retaining stocks with increasing volatility, from the lowest in portfolio 1 to the riskier in portfolio 10. In exhibit 1 we show the Sharpe ratio (risk-adjusted return in excess for the risk free rate) for each of these portfolios.

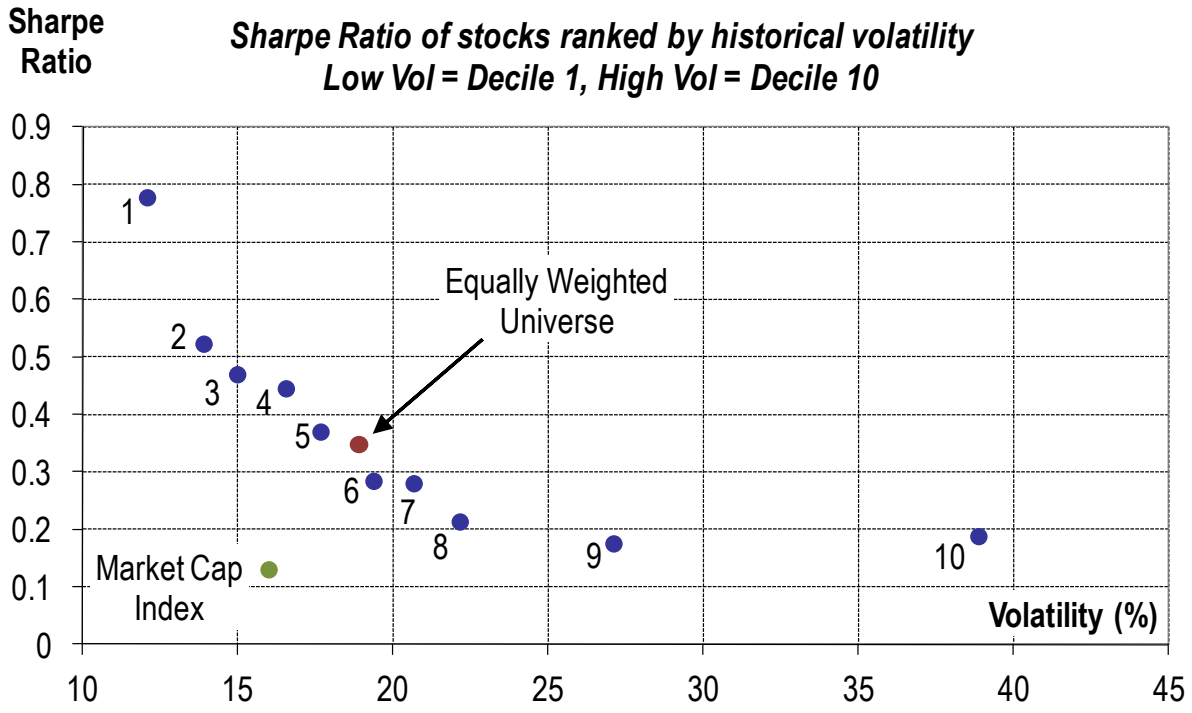

Exhibit 1: Sharpe Ratios for equally-weighed decile portfolios of stocks ranked by historical volatility based on the previous three years of weekly returns. Monthly re-balancing. Back-test period: Nov-95 to Sep-09. The investment universe is the MSCI World index and returns are in USD. Datasource: BNP Paribas Investment Partners, FactSet, MSCI.

The results show very clearly that the portfolio with the lowest volatility stocks produced the largest Sharpe ratio whereas the two portfolios with the most volatile stocks, 9 and 10, produced the lowest Sharpe ratio. Since the beta of the portfolios with the lowest volatility is also the lowest and the beta of the riskier stocks is the highest, the results are the evidence of an anomaly with the least risky stocks performing much better than predicted and conversely for the riskier stocks.

Indeed, both stocks with low volatility or low beta show this behaviour and that is, in great part, because low volatility stocks and low beta stocks are pretty much the same stocks.

Michael Jensen, Fisher Black and Myron Scholes (1972) challenged the CAPM underlying assumptions and showed what CAPM would look like when investors do not have access to unlimited leverage or cannot borrow at the risk-free rate. Their results indicated that the relationship between beta and realized return should indeed be flatter than predicted by CAPM. But still, empirical evidence for the low risk anomaly does seem much stronger than what a CAPM model with limited access to leverage alone predicts.

The evidence for the low risk anomaly has been mounting recently. There are now a large number of papers confirming the low risk anomaly forty years after it has been first reported by Haugen and Stein. Examples include Nardin Baker and Robert Haugen (1991), Jason Karceski and Josef Lakonishok (1999), Ravi Jangannathan and Tongshu Ma (2003), Roger Clarke, Harinda De Silva and Steven Thorley, (2006) and Malcolm Baker, Brendan Bradley and Jeffrey Wurgler (2011). Evidence of the low risk anomaly in the global equity markets include David Blitz and Pim van Vliet (2007), Raul Leote de Carvalho, Xiao Lu and Pierre Moulin (2012, 2012-a), David Blitz, Juan Pang and Pim van Vliet (2012) and most recently Nardin Baker and Robert Haugen (2012).

The few research papers showing evidence against the low risk anomaly have been shown to suffer from either methodological choices, e.g. Fangjian Fu (2009) who includes the forecasted period in the estimation of volatility, or survivorship bias, e.g. Lionel Martellini (2008) who used an universe with only those very few stocks which were present throughout the entire period of his back-test and excludes, therefore, companies that either went bankrupt or new entrants created during the period.

The behavioural factors behind the low risk anomaly

There are several behavioural factors behaviouralists have been putting forward in order to explain investor irrational behaviour behind this market inefficiency. These factors should explain why the demand for riskier stocks is higher than expected, justifying their lower returns than predicted by their CAPM-beta, and converse for low risk stocks.

For Nicholas Barberis and Ming Huang (2008), for example, investors perceive stocks as lottery tickets which can explain why risky stocks are overpriced. Robert Haugen explains

higher demand for risky stocks from the fact that they get much more attention and coverage from media an analysts than low risk stocks. Indeed, risky stocks have naturally more interesting stories to be presented in the media or by analysts than low risk stocks. The large price movements of risky stocks and their causes are likely to be much more entertaining and exciting than the slow price movements of low risk stocks.

David Blitz and Pim van Vliet (2007), Eric Falkenstein (2009) and Malcolm Baker, Brendan Bradley and Jeffrey Wurgler (2011) emphasized the fact that investors benchmarked against the market capitalization index, with the investment objective of producing out-performance for a given limited budget of tracking error risk, i.e. limited deviation in risk terms away from the market capitalization portfolio, will find low risk stocks unattractive. That is because despite producing the highest risk-adjusted returns, low risk stocks have the lowest exposure to the market capitalization portfolio as measured by their CAPM-beta. When they are included in benchmarked portfolio they generate huge amount of tracking error and quickly use all risk budget available. Many low risk stocks have such a large tracking error against the market capitalization portfolio that must be excluded from the start. The consequence is that the entire active management industry with mandates to out-perform the market capitalization benchmark index has been creating a natural demand for riskier stocks while shunning low risk stocks completely from their portfolios. This huge offer-demand imbalance is one of the most convincing factors that can explain the low risk anomaly. Nardin Baker and Robert Haugen (2011) also show that the compensation structure of those same fund managers is option-like in the sense that risk-taking rewards whereas non-risk taking does not. The asymmetry of the variable compensation of managers is such that this compensation increases with the level out-performance over the market-capitalization index but fund managers are not asked to pay back when under-performing. That creates a natural demand for risky stocks, which have higher exposure to the market and are therefore more likely to out-perform when markets are rising, even if on an absolute basis they offer the poorest risk-adjusted returns as it has been demonstrated empirically. One last factor was proposed by Michael Jensen, Fisher Black and Myron Scholes (1972) and later discussed in more detail by Fisher Black (1993). They showed that a limited access to leverage does flatten the relationship between risk and return. Enrico De Giorgi and Thierry Post (2011) extended this reasoning to show that short-selling restrictions also distort the risk-return relationship in a non-linear concave fashion.

Equity risk based strategies: the first generation

Equity risk-based strategies are a particular type of active strategies where the deviations to the market capitalization portfolio are determined from systematic quantitative approaches which rely only on risk views to manage risk and increase diversification. Thus, these strategies do not require any explicit stock return forecasts. Like any active strategy, the underlying portfolios must be periodically rebalanced to take into account drift and changes in risk views. Obviously, being active strategies, their performance and risk should depend at least to some extent on the active choices of re-balancing frequency and risk model used.

The most basic of such strategies is based on the equally-weighed (EW) portfolio which simply follows the principle of not putting all your eggs in one basket. The portfolio invests the same exact amount in each stock. It makes sense as an investment proposal if we believe that neither stock returns nor risk can be forecast. For a universe of *N* stocks, the EW portfolio weight allocates the same dollar amount to each stock:

$$w_i = \frac{1}{N}$$

It is clear that, when compared to the market-capitalization portfolio, the EW portfolio will overweight small-cap stocks and underweight large-cap stocks. The larger the dispersion of capitalisation of stocks the larger the difference between the EW and the market-capitalization portfolio.

The equal-risk budget (ERB) strategy invests in portfolios with the same risk budget for each stock (which is defined as the product of the stock's weight to its volatility). Risk is equally distributed among the stocks and hence riskier stocks get smaller weights. This can be seen as an extension of EW if we can trust volatility forecasts. If sigma σ_i is the volatility of stock *i* then the risk budget $w_i \times \sigma_i$ allocated to each stock is the same for each stock in the ERB portfolio, and the weight of stock *i* is:

$$w_i = \frac{1/\sigma_i}{\sum_j 1/\sigma_j}$$

We can think of the ERB as an EW portfolio tilted in favor of low risk stocks and away from high risk stocks. Therefore, the ERB portfolio is not only overweight small-cap stocks relative to the MC index, it is also overweight low volatility stocks.

If correlations are also taken into account, then we can think in terms of equal-risk contribution (ERC), where the contribution to risk from each stock is the same. Unlike the risk budget, the contribution to risk (defined as the product of the stock's weight to its marginal risk[3]) also takes into account the impact of correlations. The contribution to portfolio risk from two stocks with the same volatility but different correlations is higher for the stock with higher correlations and hence it gets a smaller weight in ERC. The stock weights cannot be written in a closed form but, as proposed by Sébastien Maillard, Thierry Roncalli and Jerome Teilietche (2008), can be found when minimising the sum of the square of the difference between the contribution to risk of any two stocks:

$$w^* = \arg\min \left(\sum_i \sum_j \left(w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2 \right) \quad \text{u.c.} \quad \sum_i w_i = 1 \text{ and } w_i \geq 0$$

The ERC portfolio can be seen as an ERB portfolio tilted towards the stocks less correlated with other stocks. It is not difficult to show that there are multiple solutions to the equation above unless the solution is constrained to be positive, i.e. long-only portfolio. Sébastien Maillard, Thierry Roncalli and Jerome Teilietche (2008) proved that the ex-ante volatility of the ERC portfolio is always between those of the EW and minimum variance portfolios defined below. Another property of the ERC portfolio is that $\tilde{\beta}_i w_i = 1/N$ with $\tilde{\beta}_i$ the beta of stock *i* estimated against the ERC portfolio and w_i the weight of stock i in the ERC portfolio.

These three strategies, EW, ERB and ERC, assume that diversification can be achieved by equally allocating wealth or risk across the investment universe. There are, however, two other risk-based strategies which are quite different in nature.

Minimum variance (MV) invests in the portfolio with the lowest ex-ante volatility. MV is, by construction, the least risky approach to investing in equities and should deliver the lowest volatility over time. It uses volatilities and correlations as inputs and is, of course, expected to invest in stocks with the lowest volatility and low correlations. The MV portfolio is the portfolio allocation with the lowest possible ex-ante variance:

$$w^* = \arg\min\ (w'\Sigma w) \quad \text{u.c.} \quad \sum_i w_i = 1$$

The MV portfolio is likely to contain short positions. Therefore it is also appropriate to add a long-only constraint.

The maximum diversification (MD) strategy, introduced by Yves Choueifaty and Yves Coignard (2008), invests in the portfolio that maximises a diversification ratio. This ratio is the sum of the risk budget allocated to each stock in the portfolio divided by the portfolio volatility. This strategy should invest in stocks which are less correlated to other stocks. The MD portfolio is the solution to the problem of maximisation of a diversification ratio D_r defined as the ratio of the weighted average of stock volatility for the stocks in the portfolio to the actual portfolio volatility:

$$w^* = \arg\max\ (D_r) \quad \text{with} \quad D_r = \frac{\sigma' w}{\sqrt{w'\Sigma w}}$$

We may expect the solution to contain short positions. Therefore a long-only constrained version may be of more interest to investors. If all stocks have the same volatility then the MD and MV portfolios are equal.

These strategies have been known for some time. The most recent are MD and ERC. ERB is well known to asset allocators who are used to work in risk-based terms since different asset classes can have dramatically different levels of risk with fixed income the less risky and equities, commodities and high yield the riskier. Investing the same amount in different asset classes generate much different levels of risk in the portfolio, hence the importance of budgeting risk instead of dollar amounts.

Robert Haugen and Nardin Baker (1991) applied the MV strategy to the stocks in the Wilshire 5000 index from 1972 through to 1989 and found that its Sharpe ratio was higher than that of the market-cap index and its volatility lower. Roger Clarke, Harinda de Silva and Steven Thorley (2012) applied MV to large-cap US stocks using data from 1967 to 2009 and confirmed Haugen's results. They even found that the actual average returns were higher for MV than for the market-cap index.

Yves Choueifaty and Yves Coignard (2008) compared the MD strategy with MV and EW applied to US stocks in the S&P500 index and to eurozone stocks in the DJ Stoxx index over the period 1992 to 2007. Not only did they observe that MV and MD portfolios out-performed the market-cap indices with lower volatility, but they also found that the MD strategy managed to generate the highest returns with volatility falling somewhere between that of the MV strategies and that of the market-cap indices. For both the Eurozone and the US they found that the EW strategy outperformed the market-cap indices with the same level of volatility.

Paul Demey, Sébastien Maillard and Thierry Roncalli (2010) investigated the performance of the EW, ERC, MV and MD strategies applied to large-cap European stocks in the DJ Stoxx 50 index from 1991 to 2009 and found that all these strategies outperformed the market-cap index and that all, bar EW, did so with lower volatility.

In our recent paper [Raul Leote de Carvalho, Xiao Lu and Pierre Moulin (2012)] we produced what we believe to be the most striking results so far, accompanied with a full discussion of the sources of risk and returns in these strategies. We tested these risk-based strategies when applied to the universe of stocks in the MSCI Index. In exhibit 2 we extended our results in the paper to include 2011 and 2012. There are no major changes arising from adding the two most recent years of data. We find that, at least since 1995, all this risk-based strategies would have out-performed the market capitalization index in absolute terms and even more on a risk-adjusted basis. That because they would have also produced lower volatility than the market capitalization index, with the exception of EW, which would have been as risky. The Sharpe ratio of all these risk-based strategies is therefore much higher than that of the market capitalization portfolio. That, in essence, is enough to catch the investor attention. According to the CAPM, where stock returns are proportional to the stock beta, as we seen before, the market-capitalization portfolio is already the most diversified and should have the highest risk-adjusted return. Thus, these empirical results contravene the CAPM and are again demonstrations of the failure of EMH.

Jan-95 to Aug-12	MSCI World index (Gross)	EW	ERB	ERC	MV long-only	MD long-only
Annualised return	3.2%	5.5%	6.0%	5.9%	6.4%	5.2%
Volatility	17.4%	17.4%	15.8%	14.1%	9.3%	10.9%
Sharpe ratio	0.18	0.31	0.38	0.42	0.69	0.47
Excess return over Market Cap		2.3%	2.9%	2.7%	3.2%	2.0%
Tracking error risk		4.7%	5.0%	6.2%	12.7%	11.7%
Information ratio		0.49	0.58	0.44	0.25	0.17
Beta		0.97	0.87	0.77	0.38	0.47
Correlation		96%	96%	94%	70%	75%
Max drawdown	-56%	-58%	-55%	-52%	-29%	-39%
Average annual turnover		38%	36%	42%	160%	152%
Average number of stocks	1729	1729	1729	1729	126	107

Exhibit 2: results from the back-tests of different risk-based strategies applied to the investment universe defined by the MSCI World Index. Returns in USD. Jan-95 to Aug-12. Datasource: BNP Paribas Investment Partners, FactSet, MSCI.

MV does produce the lowest volatility, as expected. This is also a sign that risk models do have predictive power and work better than many believe. MD has the second lowest ex-post volatility. However, MV and MD had a high tracking error risk against the market cap portfolio and invested in relative concentrated portfolios of about 120 stocks out of an average 1700 in the MSCI World index. The other three strategies, EW, ERB and ERC, invested in the entire investment universe and had a lower tracking error. Turnover of MV and MD was comparable and much higher than that of the other three. MV and MD present themselves as truly defensive strategies with a very low beta of 0.5 or less.

MV and MD have the smallest drawdown in absolute terms. However, and even if they seem to outperform the market cap index in the long-term, they can also underperform in shorter periods by as much as 50% and 40%, respectively. This was the case in the late 1990s when it would have been difficult to beat the bull market. All these risk-based strategies would have generated positive returns over that period but would have failed to outperform the market cap index.

We have also extended to 2012 our analysis of correlations of excess returns of these risk-based strategies over the market-capitalization index originally presented in our recent paper

[Raul Leote de Carvalho, Xiao Lu and Pierre Moulin's (2012)]. This update can be found in exhibit 3.

	EW	ERB	ERC	MV long-only	MD long-only
EW	100%	88%	70%	23%	32%
ERB		100%	93%	57%	61%
ERC			100%	81%	84%
MV long only				100%	96%
MD long only					100%

Exhibit 3: correlation of excess returns over the market capitalization index for the five risk-based strategies applied to the stock universe defined by the MSCI World Index. Jan-95 to Aug-12. Datasource: BNP Paribas Investment Partners, FactSet, MSCI.

The results clearly show that EW, ERC and ERC are not too different from one-another. In particular there is hardly any difference between ERB and ERC showing that impact of correlations is small. Similarly, MV and MD are surprisingly highly correlated. The results of this table can actually be explained if stocks with low volatility are also stocks with the lowest correlations, as it is in fact the case for the stocks in the MSCI World index. When this is not the case then MD can turn out to be quite different from MV as recently showed by Roger Clarke, Harinda de Silva and Steven Thorley (2012) who investigated the behaviour of MV, MD and ERC applied to the very broad US stock universe as defined by the CRSP database. For this universe which includes many very small capitalization stocks with very low correlations but high volatility they find that MV and MD are no longer comparable. MD has even higher volatility than MV and lower Sharpe ratio than the market capitalization index.

There has been several attempts to explain the risk and returns of risk based strategies in terms of their exposure to the market (through beta), and also to value stocks, small-cap stocks and momentum stocks. Since all these generate abnormally higher returns there was hope that exposures to these factors could offer the explanation of the anomalous high returns of risk-based strategies.

Several failed attempts were conducted. Yves Choueifaty and Yves Coignard (2008) regressed the returns of the EW, MV and MD strategies over the market-cap index returns for

the USA and eurozone against only three factors: the market-cap index excess returns over money market rates, the difference in returns between Value and Growth (S&P500 indices for the USA and DJ Stoxx Large-Cap indices for the eurozone) and the difference in returns between the smallest 30% and the largest 30% of stocks in terms of market-cap weights. They found that the regressions, with a positive intercept, did not fully explain the behaviour of these risk-based strategies. Roger Clarke, Harinda de Silva and Steven Thorley (2006) found that the MV strategy cannot be fully explained from exposures to value, small-cap and momentum.

Bernd Scherer (2010) was the first to give a more complete picture by including factors related to the low risk anomaly, but he only analysed the MV strategy. He showed that the MV strategy is essentially what he called a 'clumsy approach' to gain exposure mainly to risk-based pricing anomalies: low-beta stocks delivering higher returns than large-beta stocks, even after adjusting for beta, and low-residual volatility stocks delivering higher returns than high-residual volatility stocks, also after adjusting for beta. Scherer showed that the variation of excess returns of the MV strategy applied to US stocks can be largely explained by a five factor model which includes the market-cap index excess returns over the risk free rate, the returns of the two Fama-French factors, HML for value and SMB for small-cap stocks, and the returns of these two risk-based factors. He also showed that the variation of returns of these two risk-based factors cannot be explained by exposure to the market, to value stocks or to small-cap stocks as they are largely independent from these three.

In our recent paper [Raul Leote de Carvalho, Xiao Lu and Pierre Moulin (2012)] we further refined the approach of Scherer (2010) and extended it to all five risk based strategies applied to the universe of stocks defined by the MSCI World index. The results are striking and clearly show that all these risk-based strategies are finally not more than active strategies with very clear and simple stock tilts. In exhibit 4 we show an update of our original analysis to include two more years of data. We regressed the log excess returns of each strategy over the market capitalization index against five factors.

	EW	ERB	ERC	MV long-only	MD long-only
(Intercept)	0.000	0.000	0.000	0.000	0.000
Market Cap - Cash	-0.028	-0.135	-0.272	-0.727	-0.644
SMB	0.330	0.275	0.206	0.020	0.036
HML	0.128	0.144	0.113	-0.034	-0.044
LBMHB	0.081	0.180	0.302	0.541	0.567
LRVMHRV	-0.046	0.065	0.039	0.130	0.029
R-square	70%	75%	85%	86%	80%

Exhibit 3: Regression coefficients and r-square from the regression of the excess returns over market cap index against the five factors mentioned in the text. Jan-97 to Sep-12. Datasource: BNP Paribas Investment Partners, FactSet, MSCI.

The first factor is just the returns of the market-capitalization returns minus the US 1 month T-Bill rates as a proxy for the risk free rate. The second and third factors are the value and Size factors (HML and SMB). They are the well known Fama-French value and size factors as described on Kenneth French's web-site[4] where they use a large universe of US stocks. The factors here are re-estimated using the stocks in the MSCI World Index[2]. For the HML (high-minus-low) factor (value anomaly) we first ranked stocks into quintiles by market-cap and then, in each quintile, we ranked stocks by quintiles of book-to-market forming 25 portfolios. The returns of the HML factor are the returns to a portfolio long the stocks with the highest book-to-market in each quintile of market cap and short the stocks with lowest book-to-market. The returns to the SMB (small-minus-big) factor (small-cap stock anomaly) are built using a similar procedure in which we first ranked stocks by quintiles of book-to-market and then by quintiles of market-cap. The fourth and fifth factors are the Low Beta and Low Residual Volatility factors (LBMHB and LRVMHRV): after estimating the beta and residual volatility of each stock from two years of historical data we then used a similar approach to that described above for the HML and SMB factors, but in addition we neutralised beta. For LBMHB we first ranked stocks by residual volatility into quintiles and then by beta in each volatility quintile. The beta of the factor is neutralised by shrinking the short-leg in the factor by as much as required. The LRVMHRV factor was built using a similar approach but the stocks were first ranked by beta and then by residual volatility. Beta was also neutralised in a similar way as in LBMHB.

Exhibit 3 shows very clearly that factor tilts explain extremely well the tracking error risk and excess returns of these risk-based strategies. The very high r-squares and zero intercepts demonstrate that almost all variation if excess returns of these strategies is explained. The results also show that EW, ERB and ERC have an important tilt towards smaller-capitalization stocks. That is not surprising in particular for EW, which invests in all stocks of the investment universe and should overweight the most the smallest capitalization stocks and underweight the most the largest capitalization stock. The regression results also show that EW is not exposed to low beta stocks and is not a defensive strategy with an exposure of excess returns to the market factor close to zero. ERB and ERC put increasing emphasis on low-beta stocks and decreasing emphasis on small-cap stocks, more pronounced for ERC than for ERB, which seems intuitive. Both ERB and ERC exhibit a defensive beta, lower for ERC. We also observed a small exposure to value stocks in EW, ERB and ERC, but this seems much less important. MV and MD are explained by a strong positive exposure to low-beta stocks and a very low beta. Additionally, the MV is also exposed to low residual volatility stocks, but not MD, which is consistent with our expectations. We find hardly any exposure in MV and MD to small-cap stocks or to value stocks. It is reassuring that the factor exposures are in line with the theoretical expectations.

These results are perfectly in line with the analytical perspective recently presented by Roger Clarke, Harinda de Silva and Steven Thorley (2012). These authors use a single factor risk model defining the stock beta β_i and stock residual volatility $\sigma_{\varepsilon,j}$.

For the MV (long-only constrained) they showed that the stock weights are given by:

$$w_{MV,i} = \frac{\sigma_{MV}^2}{\sigma_{\varepsilon,i}^2}\left(1-\frac{\beta_i}{\beta_L}\right) \quad \text{for} \quad \beta_i < \beta_L \quad \text{else} = 0$$

where β_L is a long-only threshold beta and σ_{MV} is the risk of the minimum variance portfolio. Stocks are included in the portfolio only if β_i is larger than threshold β_L. The lower β_i the large the stock weight $w_{MV,i}$. The lower the residual variance $\sigma^2_{\varepsilon,i}$ the larger the weight, but cannot drive stock out of the portfolio.

For MD (long-only constrained) the portfolio stock weights are given by:

$$w_{MD,i} = \frac{\sigma_{MD}^2}{\sigma_{\varepsilon,i}^2}\frac{\sigma_i}{\sigma_A}\left(1-\frac{\rho_i}{\rho_L}\right) \quad \text{for} \quad \rho_i < \rho_L \quad \text{else} = 0$$

with ρ_L a long-only threshold beta, σ_{MD} the risk of the maximum diversification portfolio and σ_A the weighted average of volatility of stocks in the portfolio. Stocks are included only if ρ_i is larger than threshold ρ_L. The lower ρ_i the large the stock weight $w_{MD,i}$. The lower the stock volatility, the larger the stock weight. In MV it is the variance, not volatility, that drives the stock weight. The effect is therefore less strong in MD.

The same type of analysis applied to ERC yields a different structure:

$$w_{ERC,i} = \frac{\sigma_{ERC}^2}{\sigma_{\varepsilon,i}^2}\left[\left(\frac{\beta_i^2}{\gamma^2} + \frac{1}{N}\frac{\sigma_{\varepsilon,i}^2}{\sigma_{ERC}^2}\right)^{1/2} - \frac{\beta_i}{\gamma}\right]$$

with γ is a constant, N is the number of stocks and σ_{ERC} is the risk of the ERC portfolio. All stocks are included in the ERC portfolio – long-only by definition. The lower the residual variance, $\sigma^2_{\varepsilon,i}$, the larger the stock weights. Stock weights tend asymptotically to zero with higher stock beta β_i and asymptotically to a negatively sloped line for lower stock beta β_i.

These analytical results are in line with the regression analysis. They show clearly that there is no mystery as to where excess returns of these risk-based strategies is coming from and how is their tracking error committed. The low risk anomaly plays the crucial role in explaining what is going on.

In the absence of market inefficiencies like the low risk and the small-cap anomaly these strategies would produce returns in line with their level of CAPM-beta minus all transaction costs and market impact arising from their quite substantial turnover. Due to transaction costs and market impact, their performance would be an extremely poor alternative to a much simpler strategy investing in a mixed portfolio of market capitalization index and cash with exactly the same CAPM-beta.

But they are also sub-optimal strategies to profit from the low risk and small-cap market inefficiencies. None of these risk-based strategies has been developed bearing in mind the fact that they generate tilts towards particular types of stocks. The exposure to the different market inefficiencies arises almost by accident rather than by intention. And EW, ERB and ERC are not easy to implement as they are simply over-exposed to the smallest capitalization stocks in the investment universe for which there is the poorest liquidity. Because they invest

in all stocks in the universe they can also present some operational problems even if these have been easing in the recent years: it is much easier to implement portfolios with a large number of stocks now than it was even in the recent past. Their exposure to the low risk anomaly is not very strong as they are much more exposed to small-capitalization stocks. In turn, MV is almost exclusively exposed to the low risk anomaly. But the stock weights in MV are extremely sensitive to even very small changes in the variance-covariance matrix of stock returns. And this adds quite a lot of turnover with no additional benefit. The turnover reflects the extreme sensitivity of MV to the tiniest changes in the level of volatility of correlations of stocks. But when the changes in either stock volatility or correlations are small the stocks still remain low risk and therefore attractive from the point of view of the risk anomaly. Finally, MD is not even always exposed to market inefficiencies. For as long as stocks with low correlations have low volatility, as is the case in the mid- to large-capitalization investment universe, MD is exposed to the low risk anomaly. But for broader universes like the CRSP database for US stocks, MD ends up investing in very small stocks with very low correlations but high volatility.

The problem with these risk based strategies is that they were not designed to benefit from the actual source of risk and performance to which they are finally exposed and come with unnecessary layers of complexity which add no value, much by the contrary, they destroy value. This is why we believe they are first generation risk-based strategies: better can be done. In the next section we show examples of second generation risk-based strategies which we believe should focus first on selecting low volatility stocks and then on the portfolio construction, not the other way around.

Equity risk based strategies: the second generation

The low risk anomaly is different from other market inefficiencies. That is because the stocks which are of interest to investors with the higher risk-adjusted returns have much lower risk that all other stocks in the universe. Ideally, investing in a levered portfolio of low volatility stocks with the level of leverage set so that the final portfolio volatility is the same than that of the market capitalization index would produce substantial out-performance over the index. The problem is that i) the level of leverage required for that purpose is very large and ii) most investors do not accept leverage since the losses to a levered portfolio can in theory exceed the amount invested. That is one of the reasons why the low-risk anomaly exists in first place

as discussed previously. The risk based strategies presented in the previous section are long-only non-levered approaches which are exposed to low risk stock and profit from the anomaly, with MV being be most exposed. But MV is also the most defensive with the lowest beta and also has the highest tracking error risk relative to the market capitalization portfolio. Most investors are not prepared to invest in a portfolio with tracking error risk in the range of 10% annualised or more: that largely increases the risk of large out-performance but also of large under-performance against the market capitalization portfolio.

One could think that a constrained MV portfolio limiting the tracking error risk to more manageable levels could be indeed the solution to the problem. But this solution fails to address the problem of extreme sensitivity of MV to even small changes in the volatility or correlations of stocks and the very large turnover that this generates. And also, trying to constrain the portfolio in order to cap the tracking error risk is not an easy task. Most likely would have to be achieved by imposing a number of constraints to the stock, sector or country deviations against the market capitalization portfolio, which would almost likely bring in higher volatility stocks. To remain purely invested in the lowest volatility stocks, which have the highest tracking error risk, and create portfolios with low turnover and low tracking error risk against the market capitalization portfolio is therefore an impossible task.

However, we have recently demonstrated that the low volatility anomaly is even stronger when considered sector neutral rather than across the entire universe [Raul Leote de Carvalho, Xiao Lu, Pierre Moulin (2012-a)]. This dimension is extremely important when trying to reduce the tracking error risk of a portfolio invested in low volatility stocks since the lowest volatility stocks in the cyclical sectors have lower tracking error than the lowest volatility stocks in the defensive sectors. Indeed, another problem with the MV portfolio is that it tends to be overly-concentrated on the lowest volatility stocks in absolute terms, which are found in the defensive sectors like consumer staples, healthcare and utilities. Thus, the MV tends to avoid all other sectors of activity. But if the risk anomaly is observed in each sector, then a strategy which starts from a stock selection of the lowest volatility stocks in each sector and later considers the problem of portfolio construction to deal with the issue of tracking error risk has much more chances of succeeding. In fact, the *suite* of low volatility solutions proposed by BNP Paribas Investment Partners and managed by Theam does exactly that.

The BNP Paribas L1 Equity World Low Volatility fund, managed by Theam, has been implementing a low volatility strategy with low tracking error risk against the market capitalization index since April 2011. The investment universe and benchmark of the fund are the MSCI World Index which includes stocks developed countries across the world only. The fund invests in low volatility stocks from all sectors of activity and uses an algorithm to determine the stock weights so that the tracking error risk remains in the range of 5 to 6% annualised. The fund will naturally invest more in low volatility stocks with lower tracking error risk, found in cyclical sectors, than in those which tend to be selected by the MV strategy and have the lowest volatility but higher tracking error risk, found in the defensive sectors. Also, unlike typical constrained MV strategies, the fund will invest only in low volatility stocks and will therefore be much better exposed to the low risk anomaly than constrained MV strategies. The fund has a volatility which is 20% below the volatility of the market capitalization index and is defensive with a CAPM-beta in the range of 0.8. This means that the excess returns of the fund can be explained i) by the fact that the beta is below 1 and ii) by the abnormal excess returns produced by low volatility stocks in excess of what is predicted by their CAPM-beta. The contribution from the beta is variable in time: when markets fall the contribution to excess returns is positive and of the order of 20% the market performance in excess of money market rates, when the markets rise, the contribution is negative and of the order of 20% the market performance in excess of money rates. The contribution from abnormal excess returns of low volatility stocks not explained by beta is independent, by definition, of the market returns. We have found these abnormal excess returns of low volatility stocks to be extremely persistent over time and positive for most of the time. The low risk anomaly is extremely persistent. In the exhibit 5 we present a summary of the performance of the fund.

01-Apr-11 to 28-Sep-12	BNP Paribas L1 Equity World Low Volatility	MSCI World Index (Net Returns)
Return full period	20.04%	11.46%
Annualised return	12.99%	7.52%
Volatility	13.66%	15.20%
Sharpe ratio	0.89	0.44
Tracking error risk	5.86%	-
Information ratio	0.93	-
Beta	0.83	1.00
Jensen's Alpha	6.60%	-
Volatility of Jensen's Alpha	5.25%	-

Exhibit 5: summary of performance and risk of the BNP Paribas L1 Equity World Low Volatility fund. Jensen's alpha was obtained using the methodology proposed by Michael Jensen (1972) and measures the abnormal excess returns of the fund not explained by the fund's beta, which is 0.83 ex-post. All performances are in Euro and gross of fees. Datasource: BNP Paribas Investment Partners and Theam.

In exhibit 6 we show 12-month rolling returns of the fund (simulated from Jan-95 to Mar-11 and live performance from Apr-11 until Sep-12) against 12-month rolling returns of the market capitalization index in the same month. The returns are ranked by market capitalization returns from the most negative in the period to the most positive (not in chronological order). The excess returns of the fund against the market are also shown. It is clear that, with the benefit of hindsight, the fund is expected to generate a very large out-performance when the market returns are negative while capturing most of the upside. Indeed, we anticipate the fund to deliver returns essentially in-line with the market returns except for those few occasions when 12-month rolling market returns exceed 30% or so. Nevertheless, the fund returns are still expected to be largely positive in those under-performing occasions. The negative contribution to excess returns arising from the beta lower than 1 is too large to be compensated by the expected average positive contribution from the abnormal performance of low volatility stocks in excess of their beta. It is a small price to pay for a fund which is expected to protect on the downside, to out-perform the market in the medium to long-term and to deliver a lower risk the market. An important note of caution: past results are no guarantee of future results. What we have just presented will only be observed in the future if past behaviour is guidance for the future: the results were obtained

with the benefit of hindsight. For the fund to perform in line with back-tests the low risk anomaly must remain in place, i.e. low volatility stocks must continue to generate abnormal returns in excess of what is predicted by their beta and the predictability of the risk models used must remain comparable to what it achieved in the back-tests. There is no guarantee for that, just a conviction that we see no reason for that to change. Reassuring news are that the live performance of the fund since Apr-11 through Sep-12 has been well in line with what we just described and fulfilled our expectations.

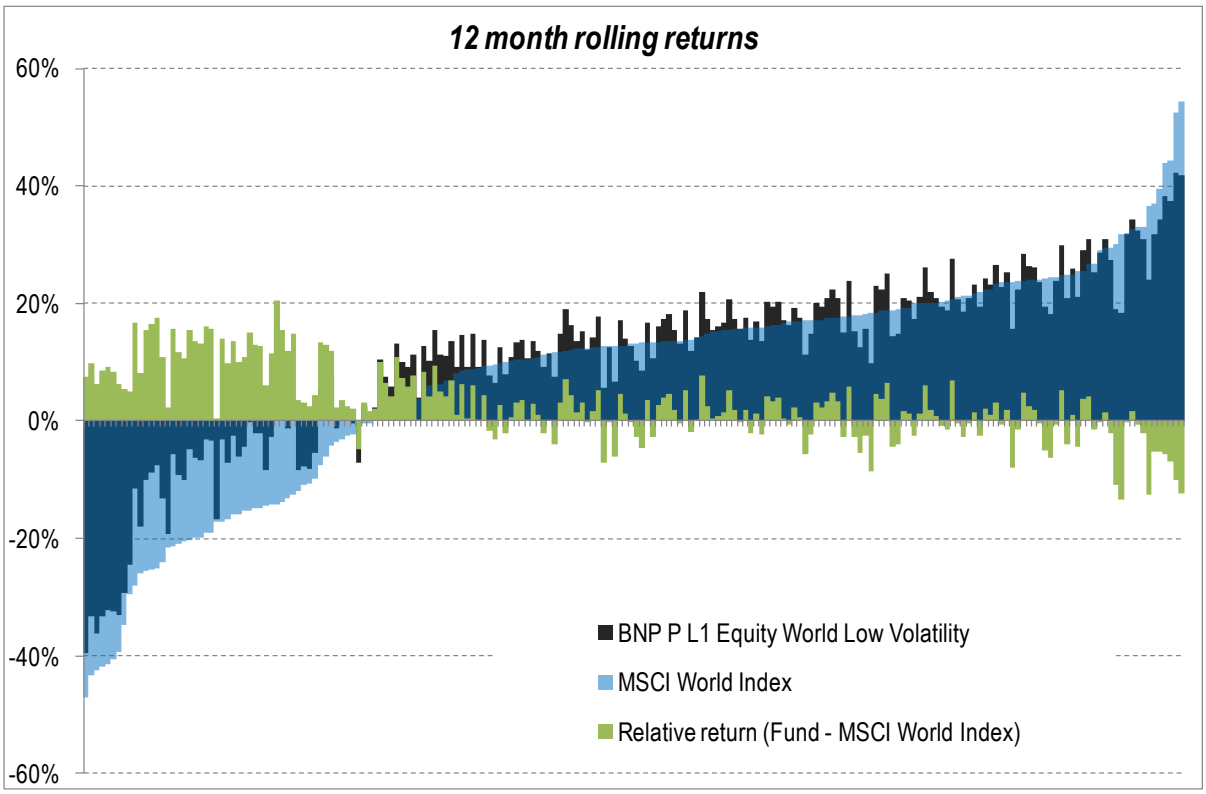

Exhibit 6: 12-month rolling returns for the BNP Paribas L1 Equity World Low Volatility fund compared to the 12-month rolling returns of the market capitalization index. The returns are ranked is ascending order by the market capitalization returns, not in chronological order. The 12-month rolling returns of the fund in excess of the market capitalization index are also shown. Returns are in USD and obtained with the benefit of hindsight for the period Jan-95 through Mar-11 and using the observed fund returns, gross of fees, from Apr-11 through Sep-12. Datasource: BNP Paribas Investment Partners and Theam.

Nardin Baker and Robert Haugen (2012) found that the low risk anomaly is observed practically everywhere, including in emerging markets. Not only the risk anomaly has been

empirical confirmed in emerging markets, we have also demonstrated that it is strong on a sector neutral basis [Raul Leote de Carvalho, Xiao Lu, Pierre Moulin (2012-a)]. Theam is about to launch an Emerging Market fund which manages a similar strategy and delivers performances and risks of comparable behaviour to what has just been described above for the BNP Paribas L1 Equity World Low Volatility fund. The fund uses stocks from the MSCI Emerging Market index universe and is expected in the fall of 2012.

Some investors may not have constraints on the tracking error risk against the market capitalization index. They may reason in absolute terms and seek the highest risk-adjusted returns. The BNP Paribas L1 Equity World Low Volatility fund does have on constraint which is to keep the tracking error risk at around 5 to 6%. This constraint tends to push the strategy more into cyclical low volatility stocks rather than defensive low volatility stocks. But when the tracking error risk is not a constraint, then a better portfolio can be constructed by removing that constraint.

The BNP Paribas Pure Low Volatility strategy is still just a concept but should also soon be implemented by Theam in a new fund expected in early 2013. This strategy invests in the stocks with the lowest volatility from each sector at global level and uses an algorithm to diversify as much as possible the final portfolio. The goal is to achieve a good representation of all sectors and stocks in sectors and profit as much as possible from the low-risk anomaly. The strategy is being independently assessed since Feb-12 by am-League (*www.**am-league**.com/en/*), a company that monitors and builds track-records for asset managers based on the model portfolios sent out by the managers. The investment universe used in the am-League version of the strategy is the Stoxx 1800, a global index not too different from the MSCI World Index in terms of composition.

In Exhibit 7 we compare simulations of the performances and risks of the strategy behind the BNP Paribas L1 Equity World Low Volatility fund, the BNP Paribas Pure Low Volatility strategy and a Minimum Variance strategy. The period used in the back-tests is Jan-95 through Dec-11. Monthly re-balancing applies. The Minimum Variance strategy chosen is that proposed by MSCI through its MSCI Minimum Volatility Index for global developed countries. Here, all approaches apply to the same universe of stocks, that of the MSCI World Index. The BNP Paribas approaches use a proprietary risk model whereas the MSCI

Minimum Volatility Index uses the Barra model, also provided by MSCI. The methodology of the index is publicly available.

Strategy Jan-95 to Dec-11	Annualised Returns	Volatility	Excess Returns	Tracking Error risk	Sharpe ratio	Information ratio
MSCI World Index	5.6%	16.0%	-	-	0.09	-
MSCI Minimum Volatility Index	7.8%	11.4%	2.2%	7.30%	0.36	0.30
BNP P Equity World Low Volatility Fund	9.6%	13.1%	4.0%	5.50%	0.45	0.72
BNP P Pure Low Volatility Strategy	12.3%	10.4%	6.6%	9.10%	0.82	0.73

Exhibit 7: simulated performances and risks of three different risk-based strategies compared to the market capitalization MSCI World Index. The MSCI Minimum Volatility index is a constrained Minimum Variance strategy. The BNP P L1 Equity World Low Volatility is a strategy which targets investing in low volatility stocks to profit from the low-risk anomaly and improve Sharpe ratio while keeping the tracking error risk in the range of 5 to 6% whereas the BNP P Pure Low Volatility strategy targets investing in low volatility stocks to profit from the low risk anomaly in order to maximise its Sharpe ratio without tracking error constraints. Results are in USD and obtained with the benefit of hindsight. Datasource: BNP Paribas Investment Partners, Theam and MSCI.

The simulations in exhibit 7 show that the BNP P Pure Low Volatility strategy would have produced by far the largest possible Sharpe ratio. That is because it is free of constraints and has been designed to optimise its exposure to the low risk inefficiency. But the lack of constraints mean that investors must accept a large tracking error risk against the market capitalization index of 9% annualized. The Sharpe ratio before transaction costs, market impact and gross of fees is 0.82, much higher than that for the market capitalization index in the period at 0.09. In the long-run we would expect the market capitalization index to produce a Sharpe ratio in the range of 0.20 to 0.30, in line with very long-term equity market historical returns. The strategy in the BNP P L1 Equity World Low Volatility fund produces a lower Sharpe ratio as a consequence of the constraint to reduce the tracking error risk and keep it at 5 to 6%. That is achieved by reducing the diversification when compared to the BNP P Pure Low Volatility strategy. But the information ratio is comparable for both. Only the absolute return is also smaller in the latter at 9.6% compared to 12.3% for the BNP P Pure Low Volatility strategy. The paradox of the risk based anomaly is that would have allowed investors to earn higher returns with lower risk, at least in the past. The MSCI Minimum Volatility index suffers from the fact that is over-constrained in terms of turnover and active

weights when compared to the BNP Paribas strategies. Indeed, the turnover of the MSCI Minimum Volatility index is only 20% annual (one-way) whereas the BNP Paribas strategies reach 100 to 120% (one-way).

This raises the question of how much should an investor expect to lose to market impact and transaction costs. We use our proprietary model for market impact to estimate, with the benefit of hindsight, how much would have been detracted from the strategy returns for different levels of assets under management. The results can be found in exhibit 8.

AUM on 31-Dec-2010 USD Million	BNP P L1 Equity World Low Volatility fund	BNP P Pure Low Volatility strategy
500	0.23%	0.40%
1000	0.27%	0.48%
2000	0.32%	0.57%
5000	0.42%	0.76%

Exhibit 8: Estimated annual cost from market impact and transaction costs estimated from a proprietary impact cost model. Datasource: BNP Paribas Investment Partners.

Conclusions

Behavioural finance has long been advocating that empirical violations of EMH can be explained by *irrational* behaviour of investors. Evidence of violations to EMH and CAPM has been repeatedly reported at least for forty years and that has not been enough to turn the market efficient. Many investors continue to approach the markets irrationally creating opportunities for other investors in what is a zero-sum game. Value, size, momentum have been widely discussed by academics and explored by practitioners. The low risk anomaly, which is known for even longer, has however been ignored by practitioners until recently. That can be explained by the reasons that create this anomaly in first place, in particular that low risk stocks tend to have too high tracking error risk not tolerated by investors benchmarked against the market capitalization index. Fund managers are almost forgiven for under-performing in down markets and expected to out-perform in bull markets. They also have an interest to do so because of their compensation structure. A general appetite for risk also explains with the anomaly is so persistent.

Therefore the risk anomaly has been forgotten until recently. Risk-based strategies have recently been offer by asset managers but in a poor format. We expected these first generation risk-based strategies such as minimum variance to fall out of love in the next

strong bull market because they simply have too low beta and not sufficiently diversified stock allocations. It already happened in the past and will happen again. Strategies like equal-risk contribution, equal risk-budgeting or equal-weighting will disappoint as returns will be lower than expected due to poor liquidity of the smaller capitalization stocks. Exiting these latter three strategies is likely to be challenging in some market environments. Maximum diversification, in our view, suffers from the problem that is only weakly exposed to the low risk anomaly, and is therefore of much less interest than even minimum variance.

The suite of strategies offered by BNP Paribas Investment Partners and Theam addresses these questions. A good balance between the level of beta, not too low, and diversification is achieved in the BNP Paribas L1 World Equity Low Volatility fund. That means that it should be extremely resilient in bull markets, unlike minimum variance strategies. The diversification at sector level is of great importance to generate additional abnormal excess returns over what should be expected from the fund beta and increase the information and Sharpe ratio over other approaches.

The BNP Paribas Pure Low Volatility strategy will be less resilient in bull markets due to a much more defensive beta, i.e. much lower than 1, but since it captures the low risk anomaly in a more diversified way than the typical minimum variance strategies is also expected to perform much better than those of average, including in bull markets. This strategy is particularly interesting for its high Sharpe ratio, at least observed in the past with the benefit of hindsight.

The specialists of BNP Paribas Investment Partners and Theam remain committed to research and will continue to push the frontiers of finance further in search of be most suited and best designed products for our clients while always recognising that out-performing the market capitalization index, being a zero-sum game, is difficult and require staying in the lead.

Notes:

1. The semi-strong-form of EMH assumes that stock prices adjust to publicly available information sufficiently rapidly for any fundamental or price based information to be useful in predicting returns. The weak-form of EMH assumes the stock returns cannot be predicted from past stock prices but does not exclude that information not based on

historical prices, like some forms of fundamental information, may have some predictive power.

2. Due to licensing constrains, prior to 2005, we used the universe of global stocks from developed countries from the Exshare® database for which the market-cap allocation minimises the tracking risk against the total returns of the MSCI World index in USD. Therefore, the universe used prior to 2005 may not be exactly that underlying the MSCI World index and we believe that our universe is likely to contain more stocks that those in the MSCI index in the period 1995 - 2004.
3. The marginal stock risk is defined as $\partial_{w_i}\sigma(w) = \partial\sigma(w)/\partial w_i$ with w_i the weight of the stock *i*.
4. http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

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